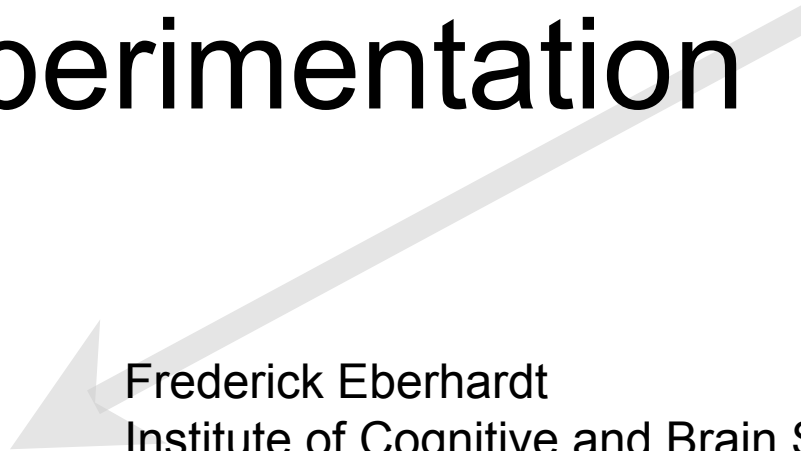
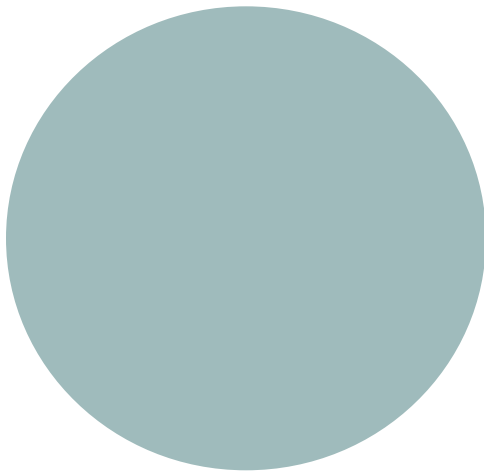
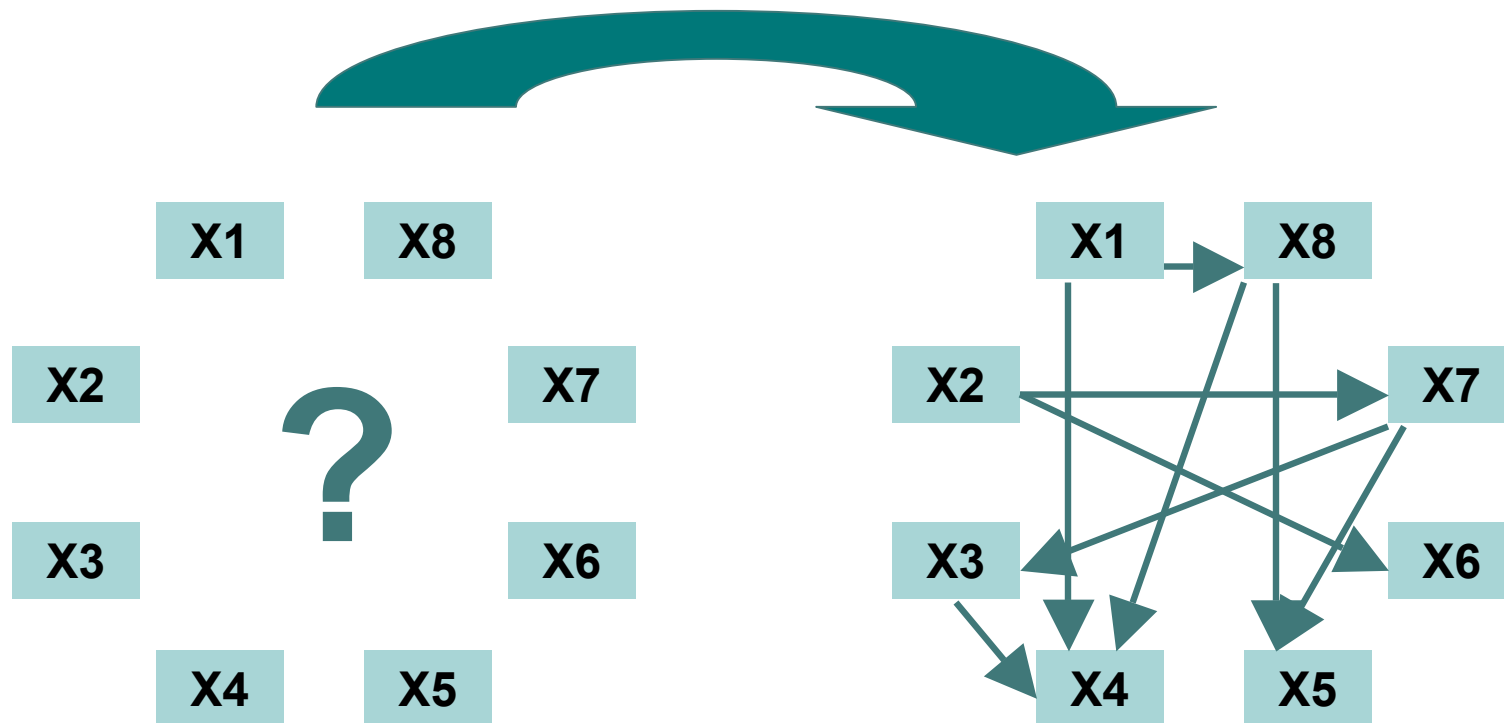
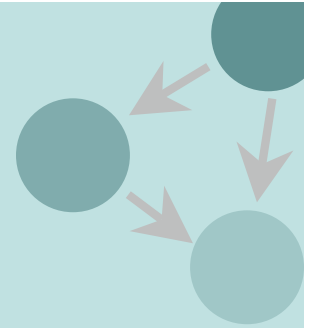


# Strategic Experimentation

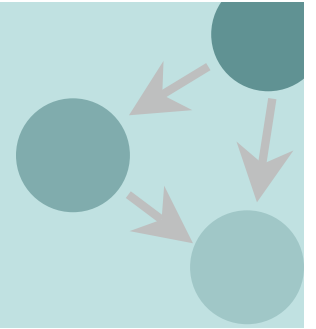


Frederick Eberhardt  
Institute of Cognitive and Brain Science  
University of California, Berkeley;  
Philosophy, Neuroscience and Psychology,  
Washington University in St. Louis  
[fde@berkeley.edu](mailto:fde@berkeley.edu)  
[www.phil.cmu.edu/~fde](http://www.phil.cmu.edu/~fde)

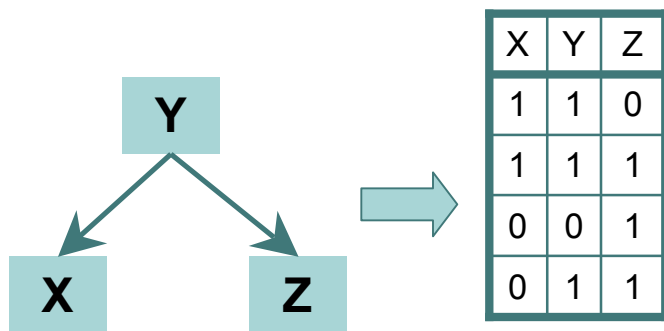
# Problem: Causal Discovery



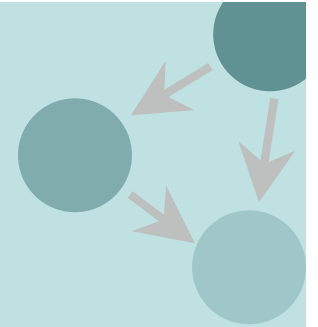
# Structure Search



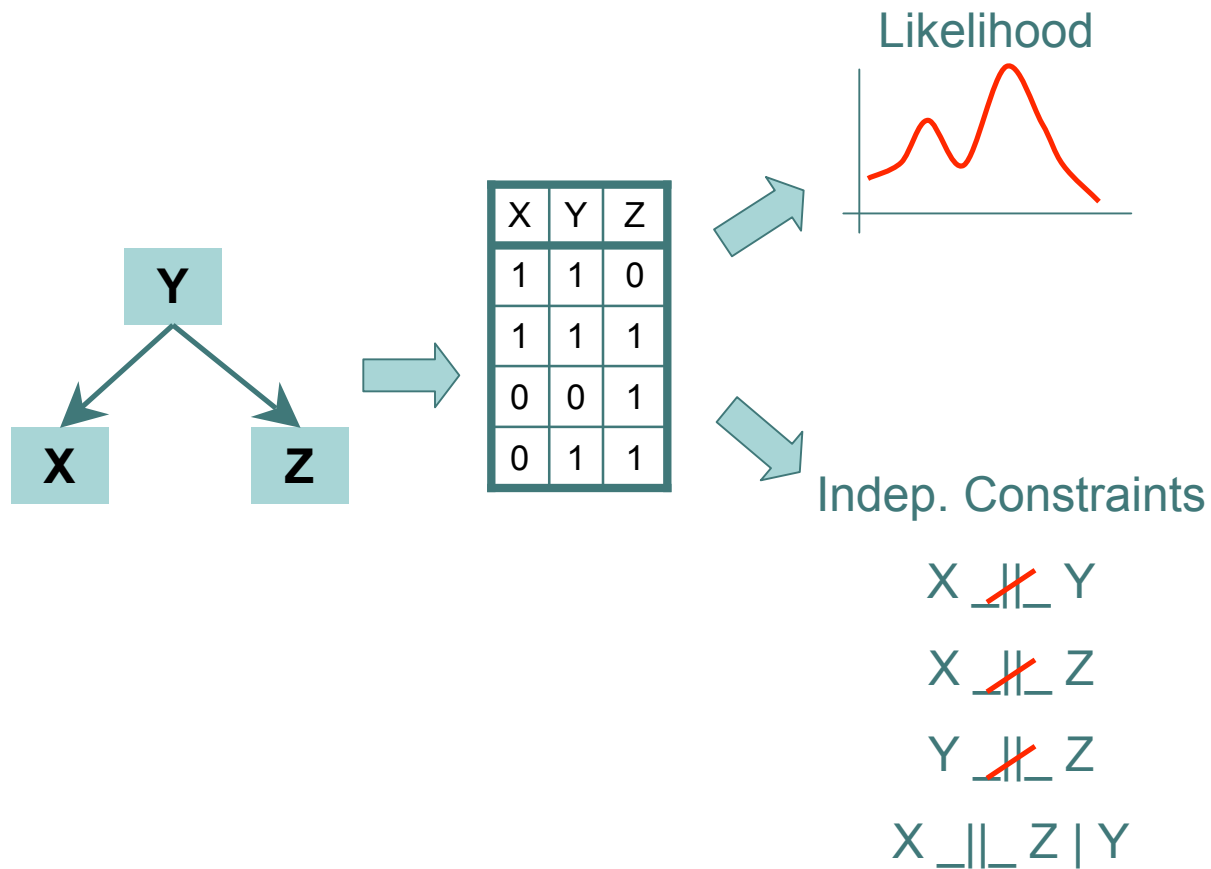
- Search in passive observational data



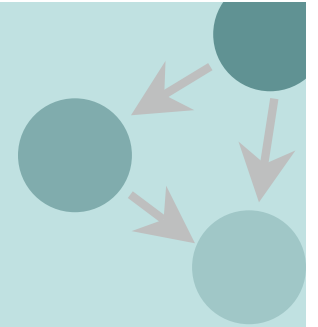
# Structure Search



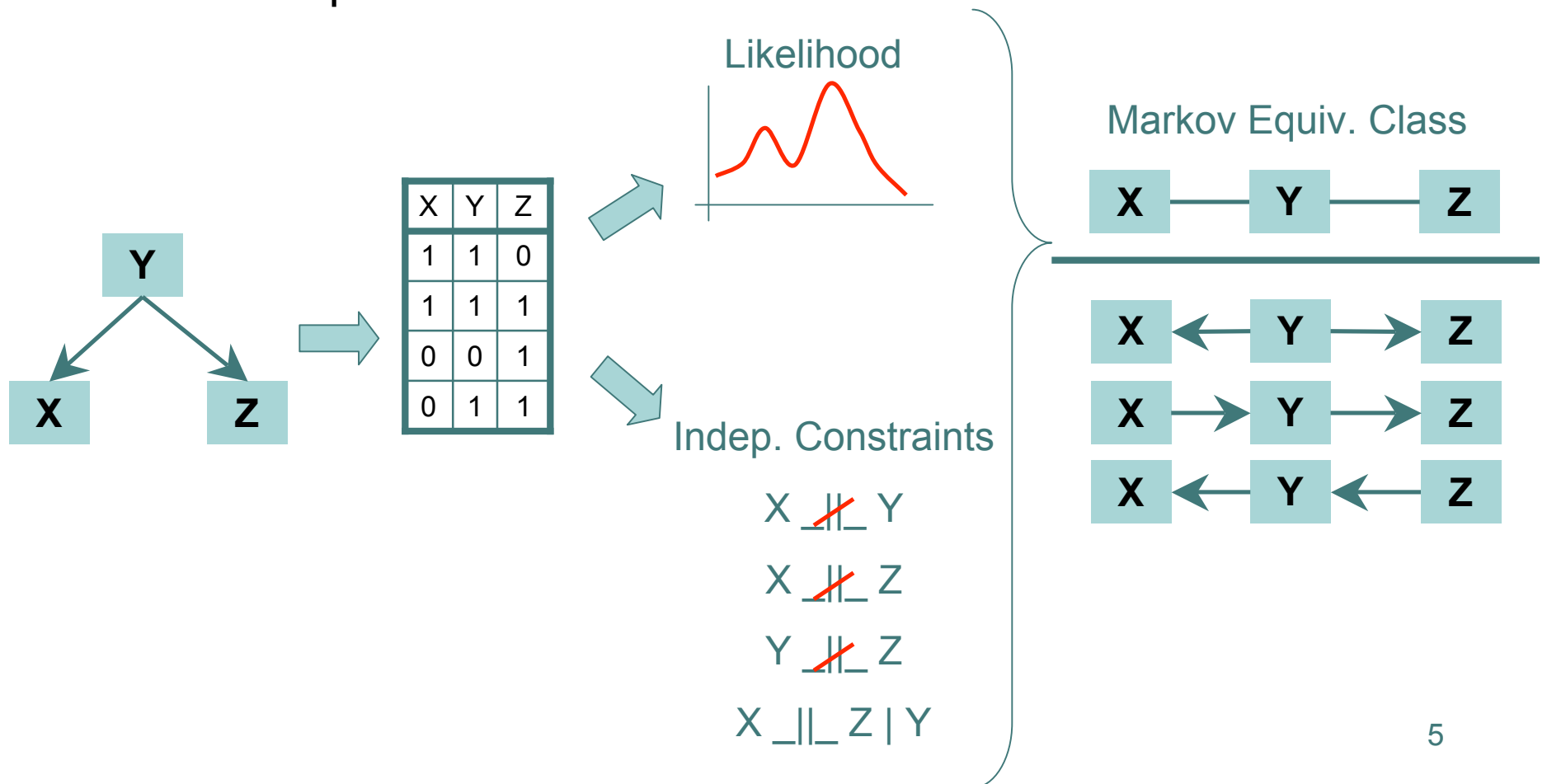
- Search in passive observational data



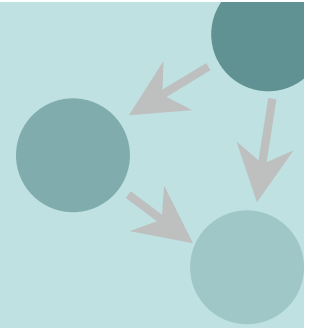
# Structure Search



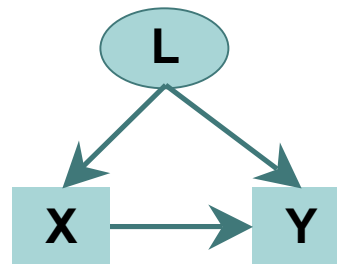
- Search in passive observational data



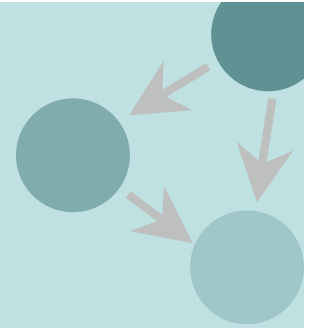
# Structure Search



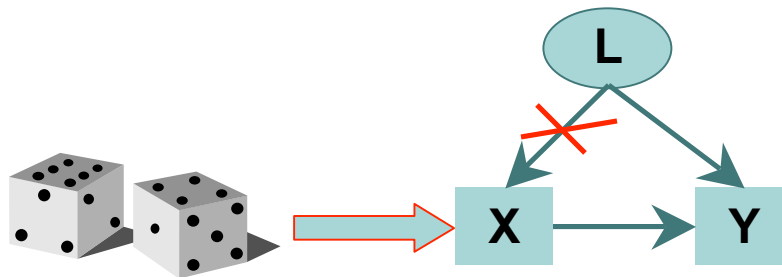
- Randomized Controlled Trials



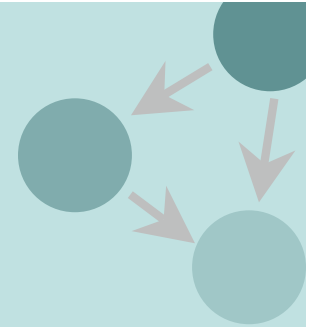
# Structure Search



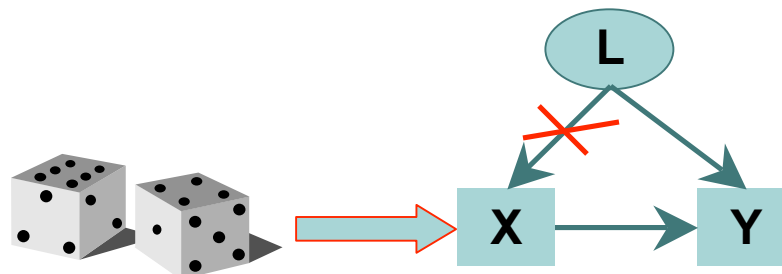
- Randomized Controlled Trials



# Structure Search

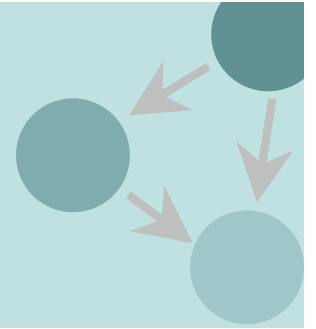


- Randomized Controlled Trials



→ Experimental Design: Potential causes and potential effects are known, search is restricted to bipartite structures

# Equivalence Classes



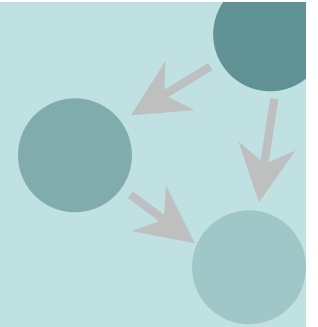
- Observational



- Interventional



# Equivalence Classes



- Observational



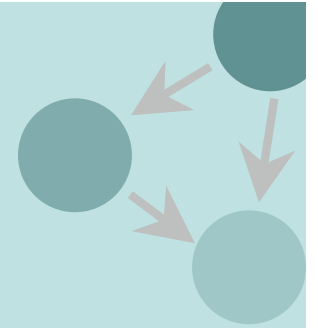
- Interventional



...



# Equivalence Classes



- Observational



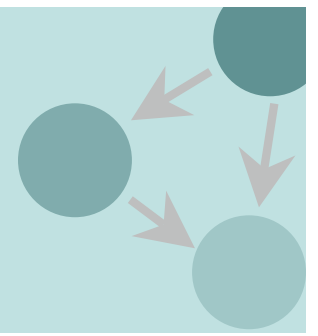
- Interventional



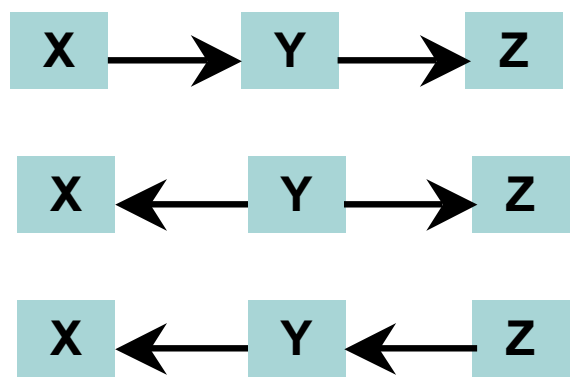
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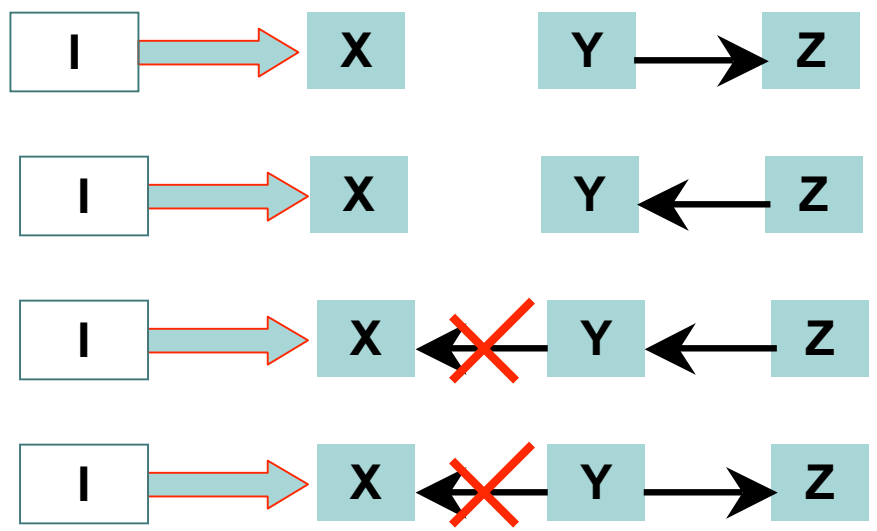
# Equivalence Classes



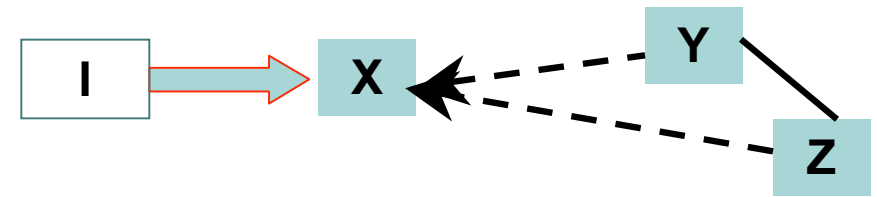
○ Observational



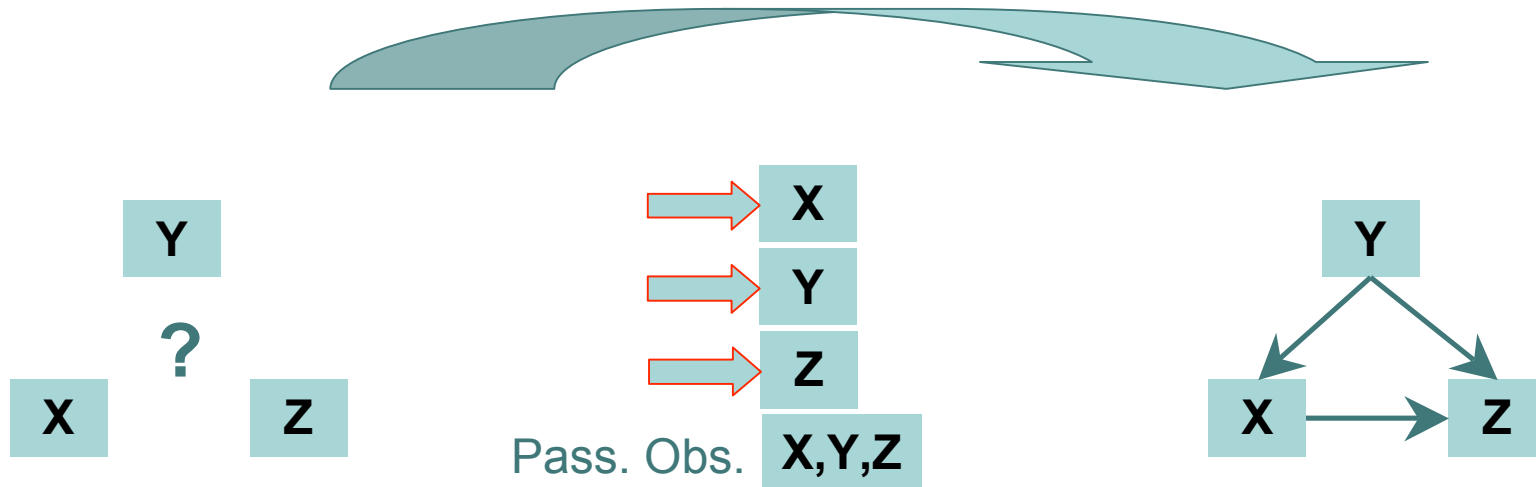
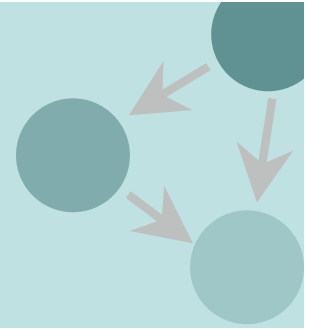
○ Interventional



...

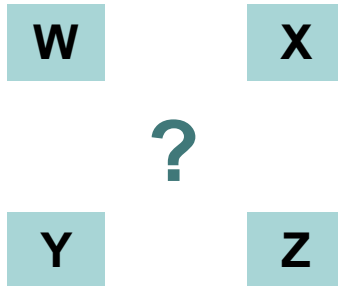
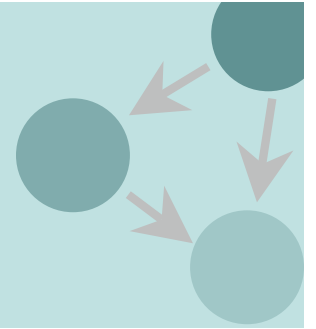


# Active Search

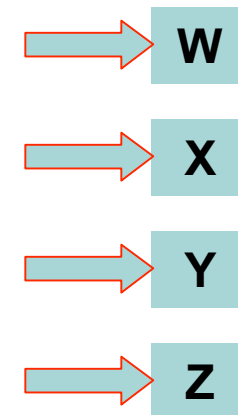
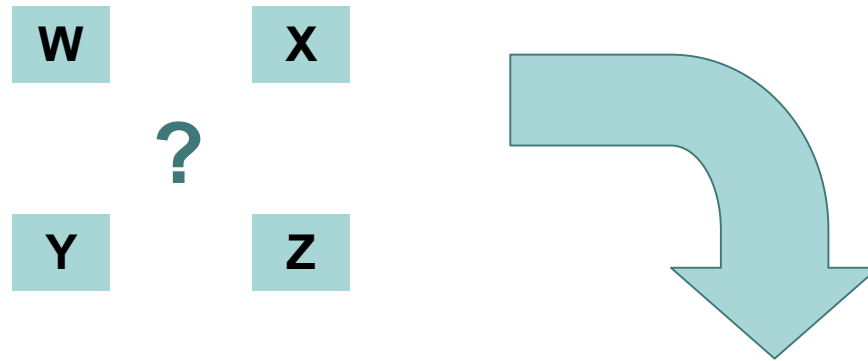
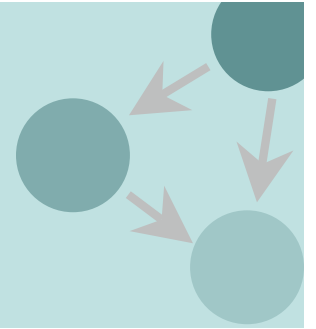


- How many (and which) experiments are necessary and sufficient to discover the causal graph among N variables?

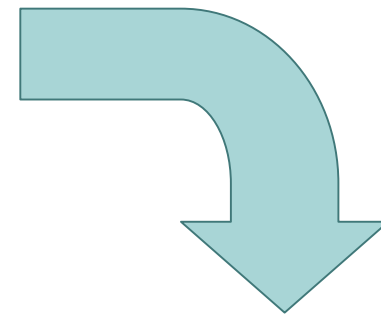
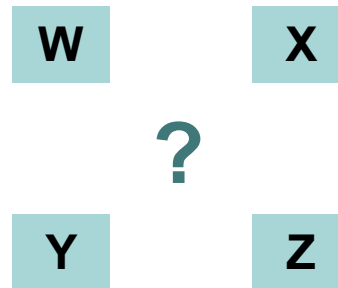
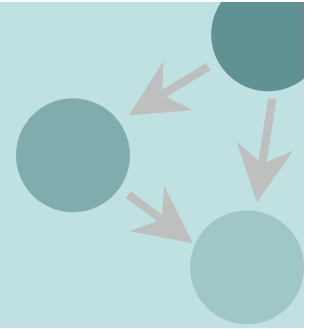
# Bayesian Active Search



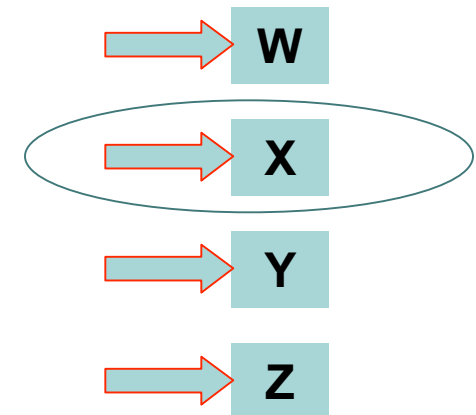
# Bayesian Active Search



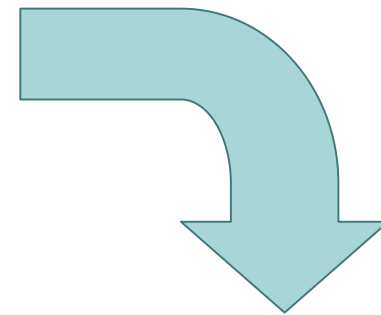
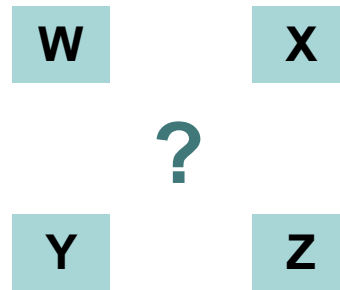
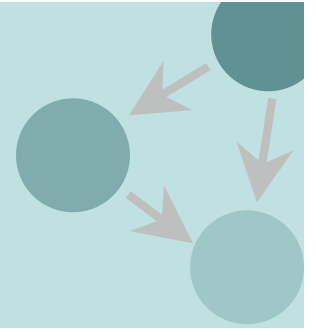
# Bayesian Active Search



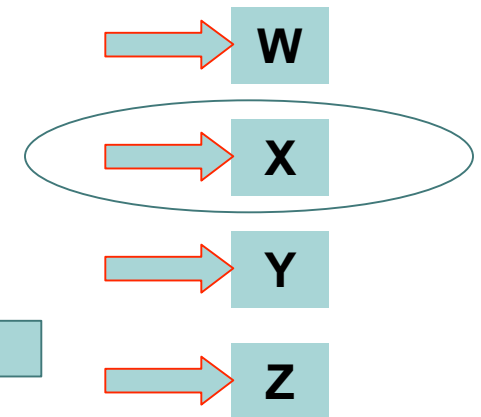
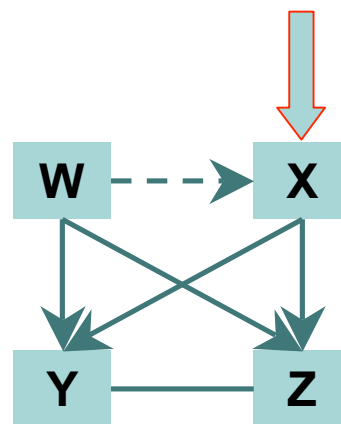
Information  
Theoretic Measure



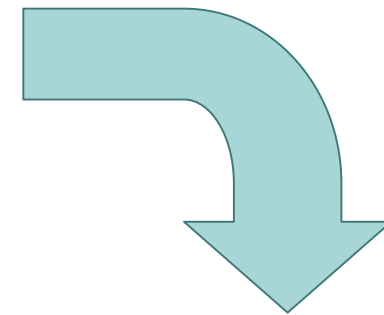
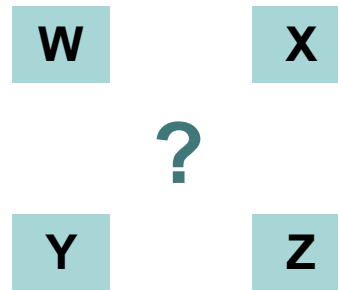
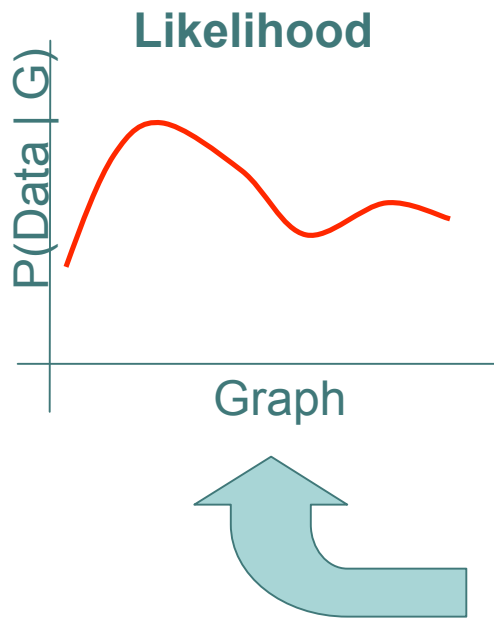
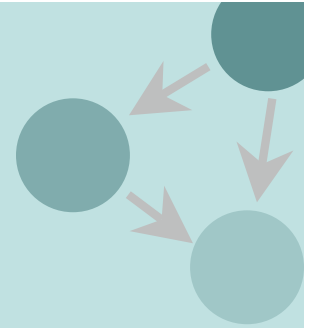
# Bayesian Active Search



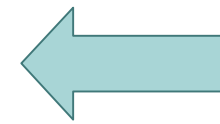
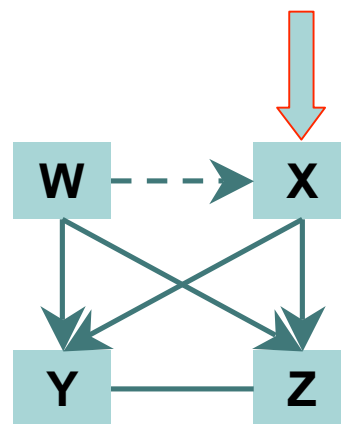
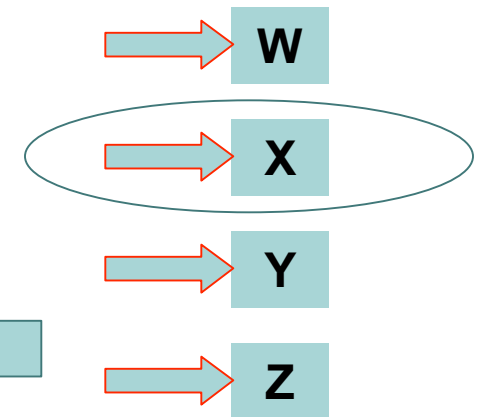
Information  
Theoretic Measure



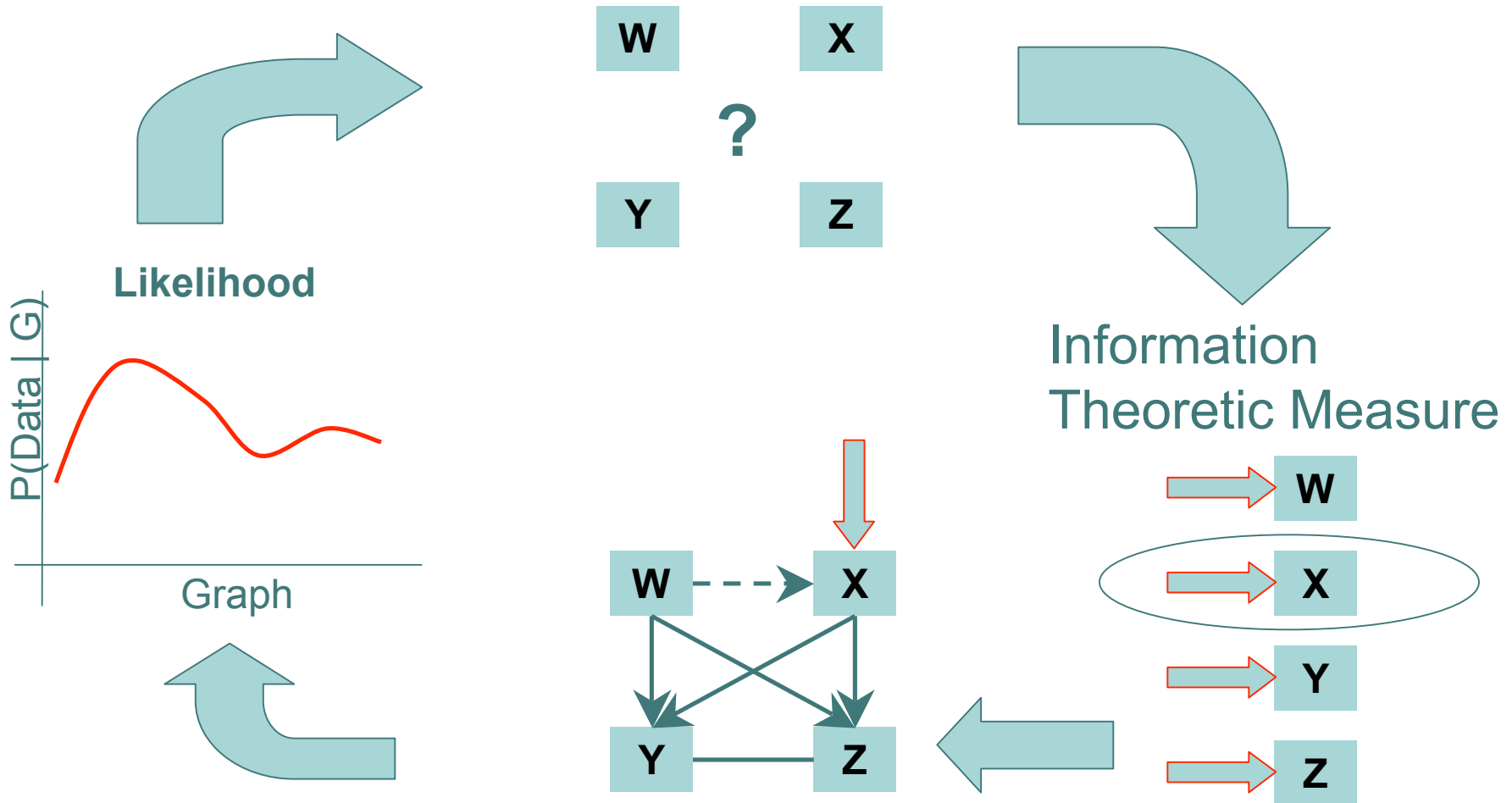
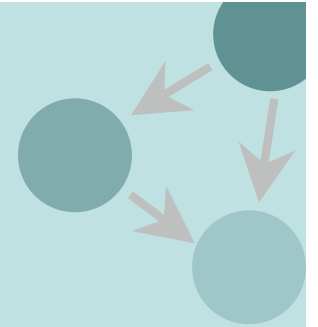
# Bayesian Active Search



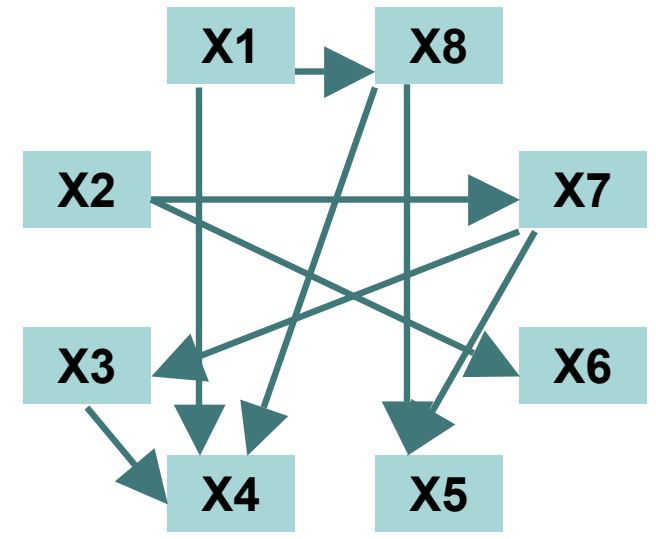
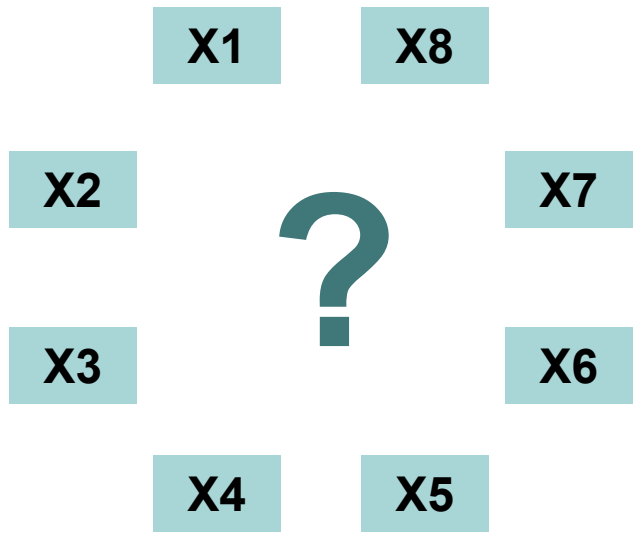
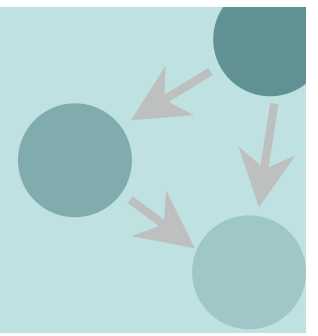
Information  
Theoretic Measure



# Bayesian Active Search



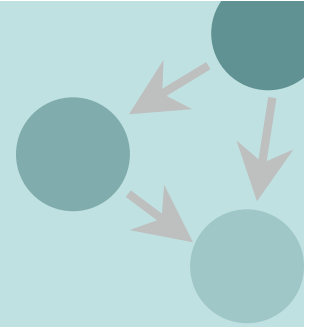
# My Problem



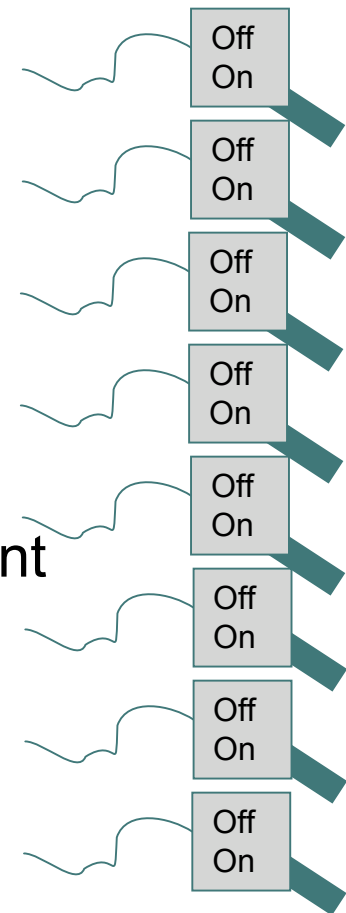
- How many (and which) experiments are necessary and sufficient to discover the causal graph among N variables?

→ **Characterization of cases that can be discovered using interventions**

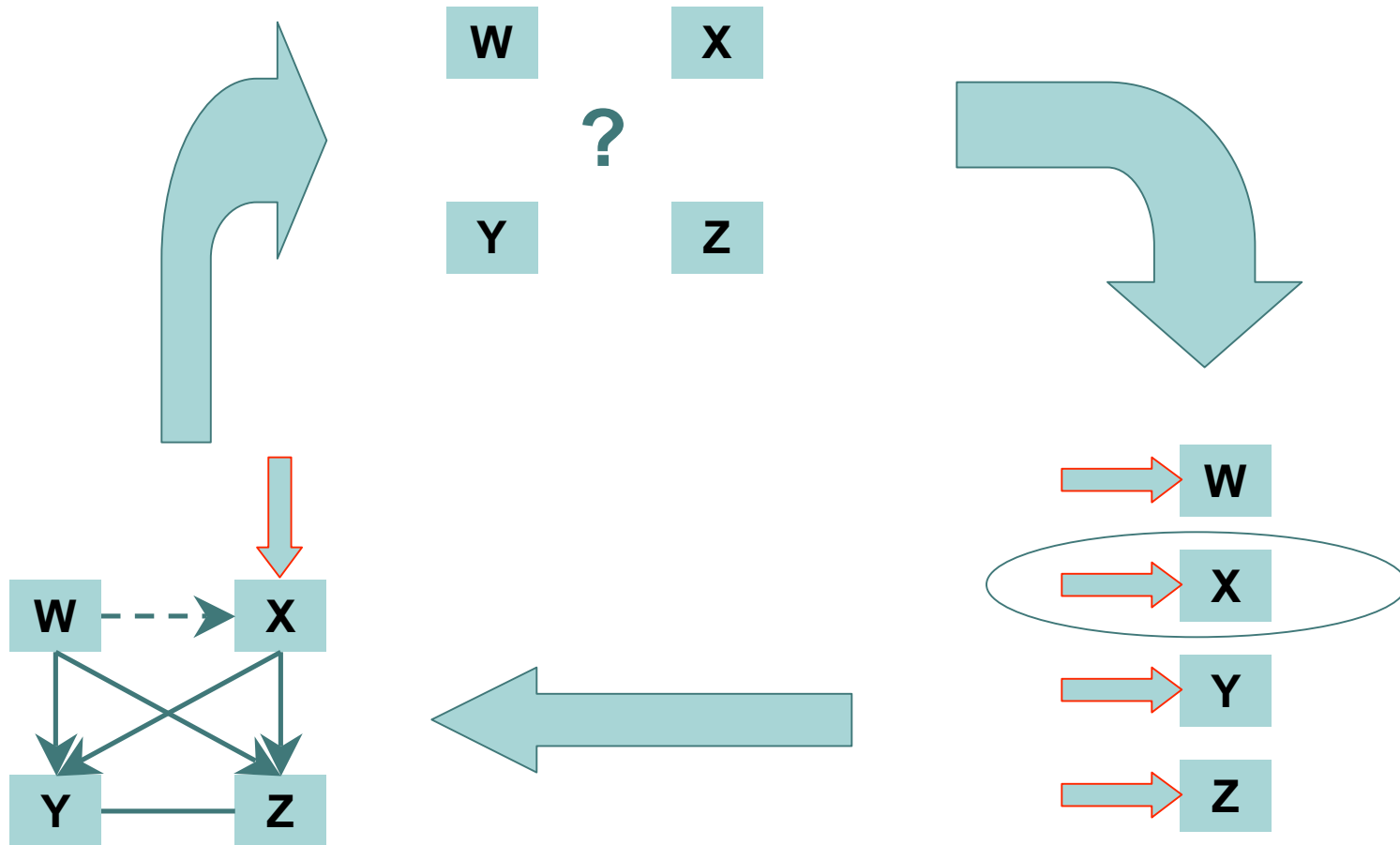
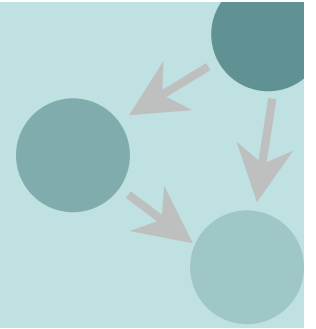
# Assumptions



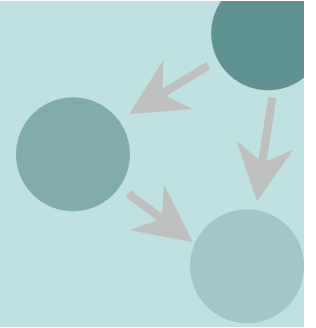
- Causal Markov
- Causal Faithfulness
- Acyclicity
- Interventions possible on every variable
- Independence Oracle
- Intervention makes intervened variable independent of its normal causes
- Single Intervention per Experiment
- Causally sufficient set of variables



# Sequence of Experiments

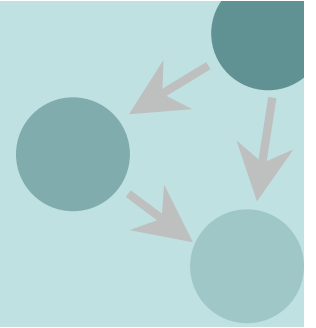


# Fixed Search Strategies



- Announce sequence of experiments BEFORE any experiment is performed.

# Fixed Search Strategies

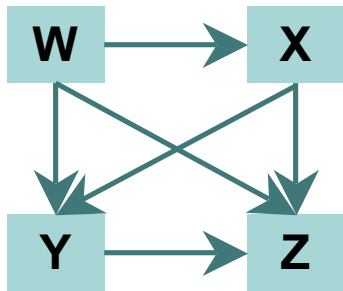
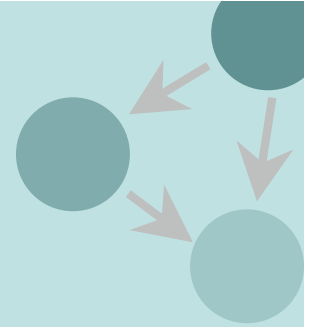


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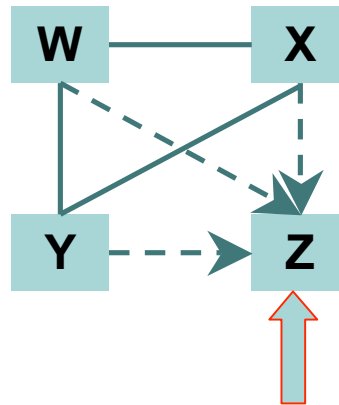
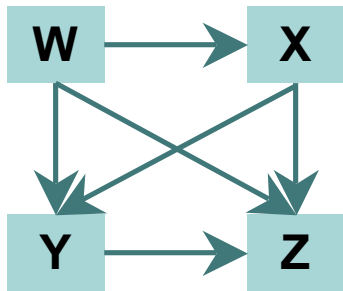
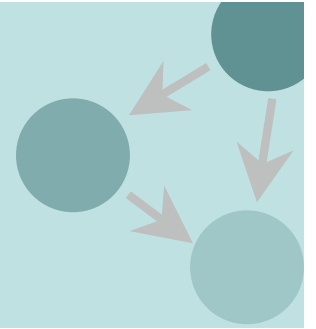
→ Variables: {W, X, Y, Z}

	Strat A	Strat B	Strat C	...
Exp 1	→ Z	→ W	→ W	...
Exp 2	→ Y	→ Z	Pass. Obs.	...
Exp 3	→ X	→ Y	→ Z	
...	...	...	...	...

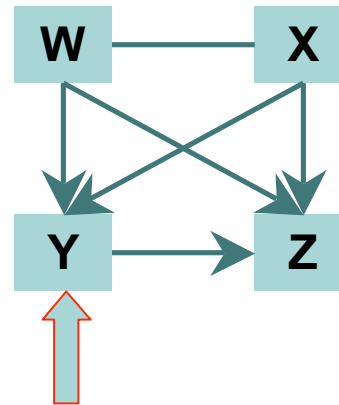
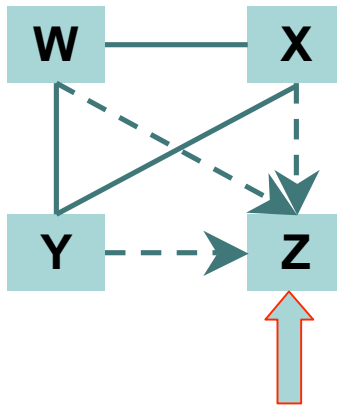
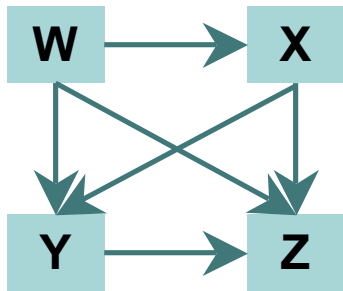
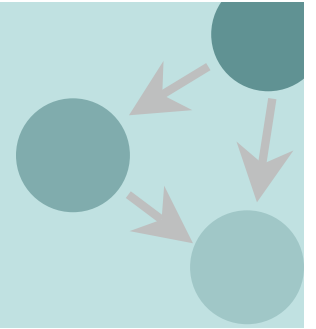
# Example: Strategy A (single)



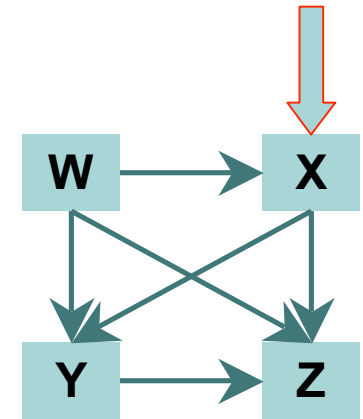
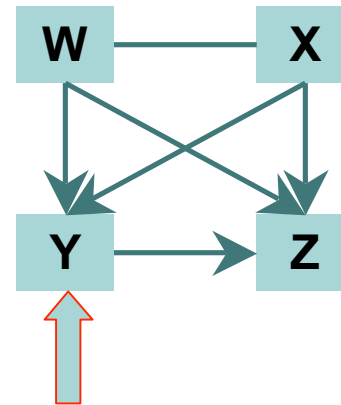
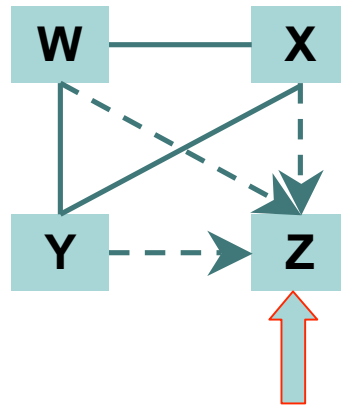
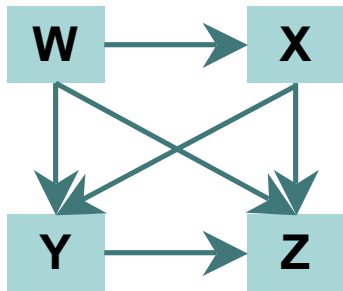
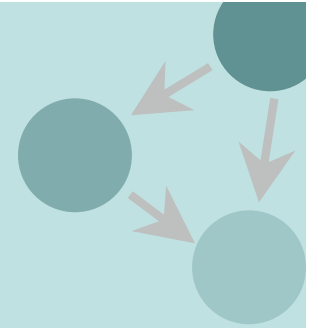
# Example: Strategy A (single)



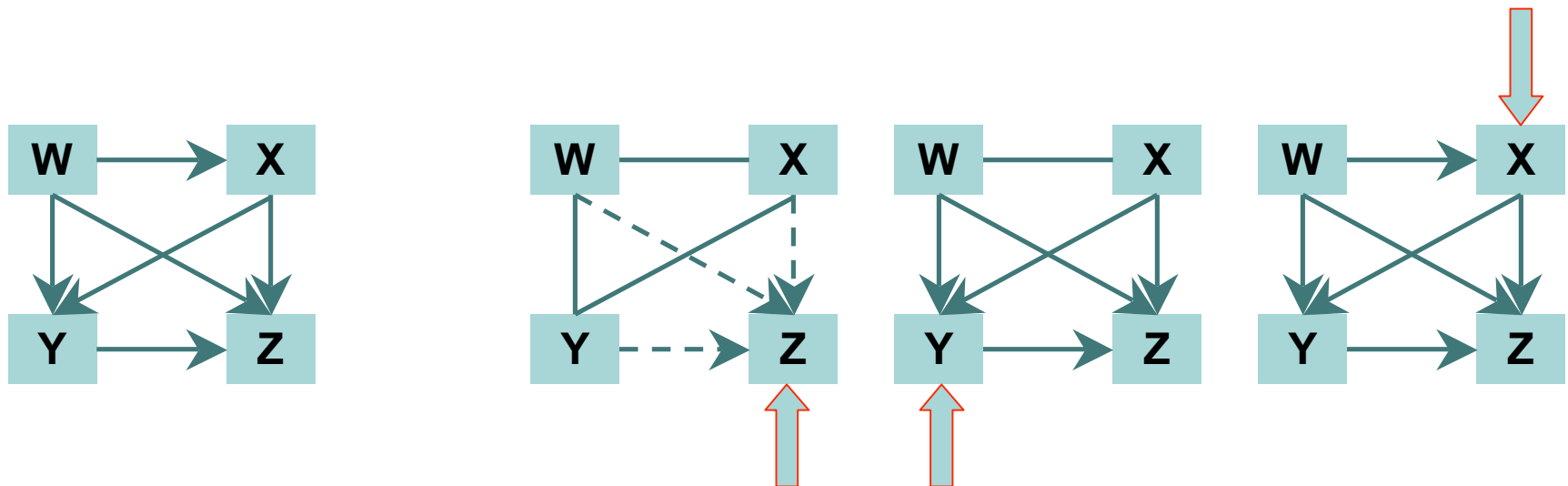
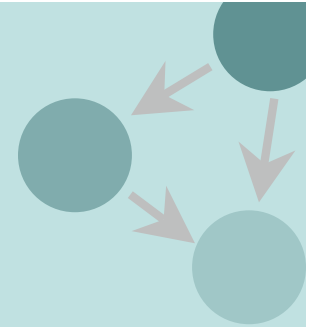
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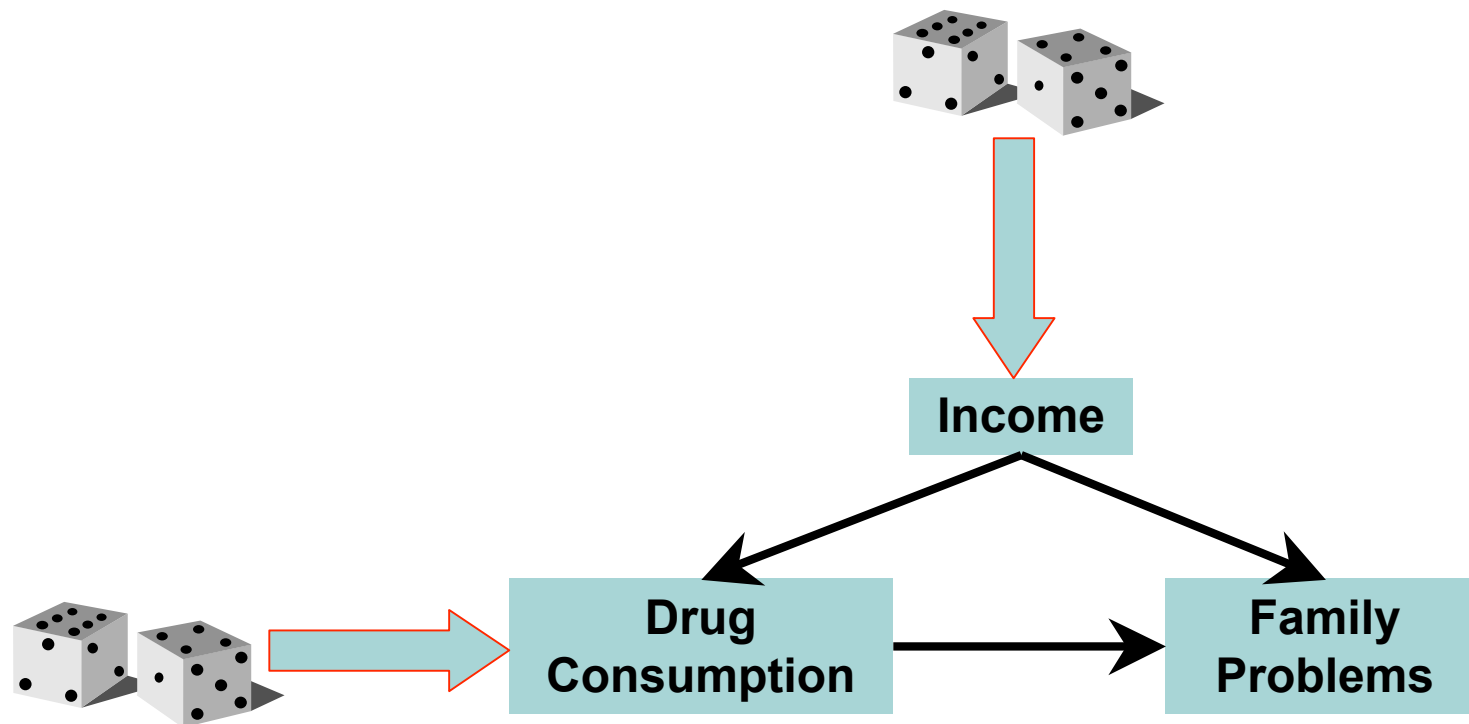
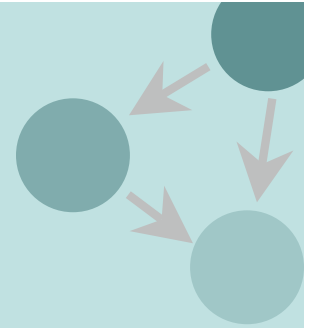


# Example: Strategy A (single)

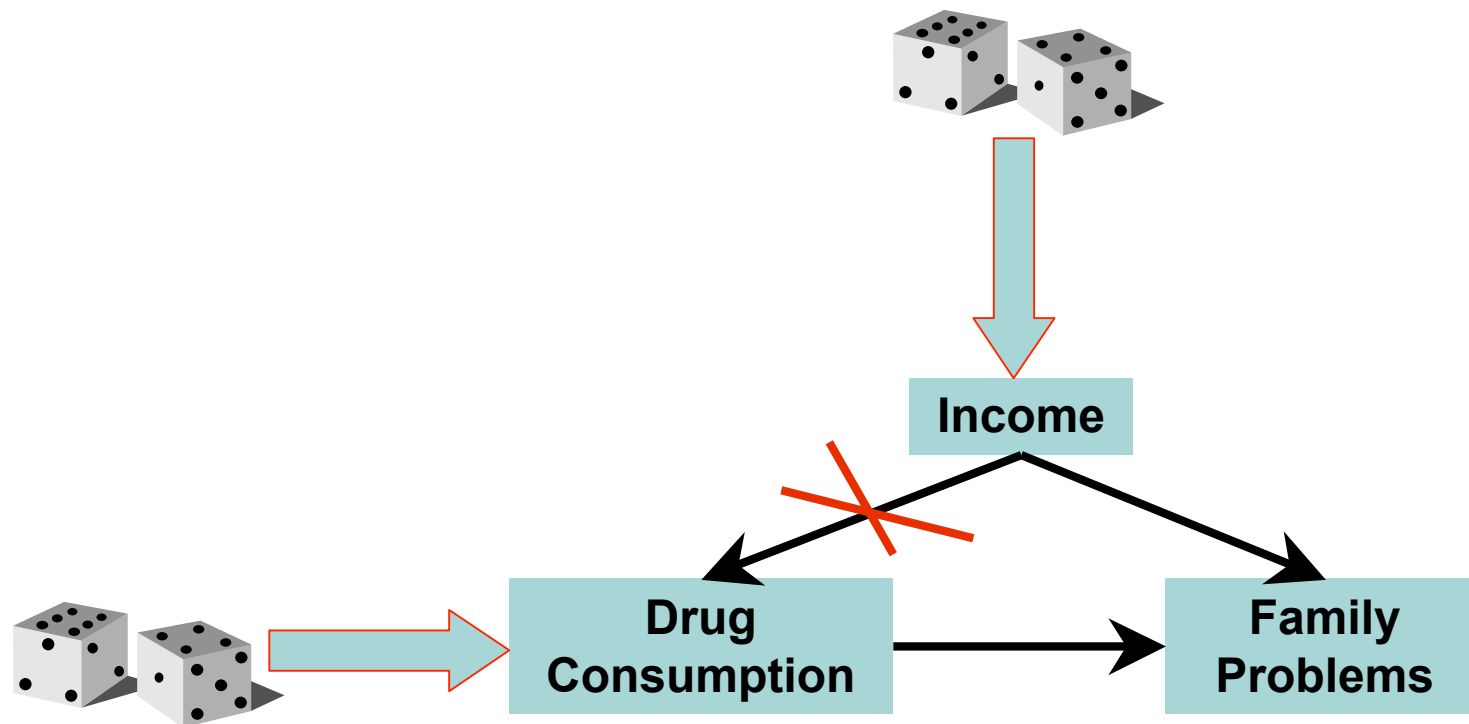
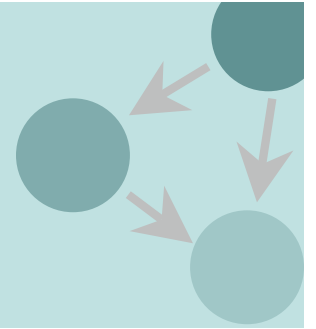


- In general: **N-1** experiments are sufficient and in the worst case necessary to discover the causal graph among N variables.

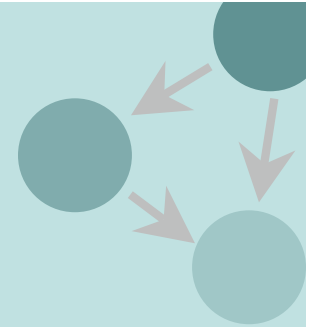
# Multiple Simultaneous Interventions



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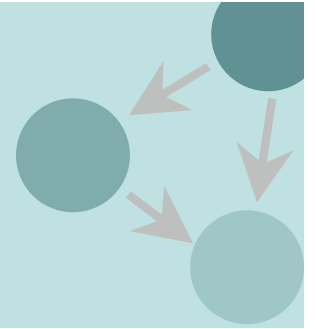
# Fixed Search Strategies



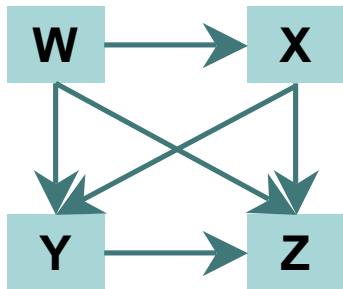
- Multiple simultaneous interventions per experiment:

	Strat A	Strat B	Strat C	...
Exp 1	 	 		...
Exp 2	 		Pass. Obs.	...
Exp 3	Pass. Obs.	 	  	...
...	...	...	...	...

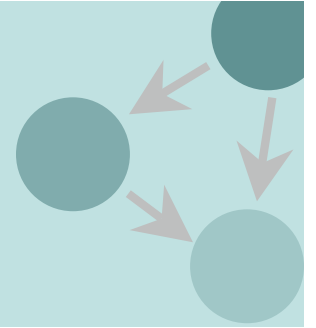
# Example: Strategy A (mult)



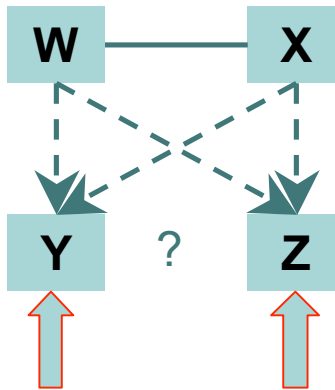
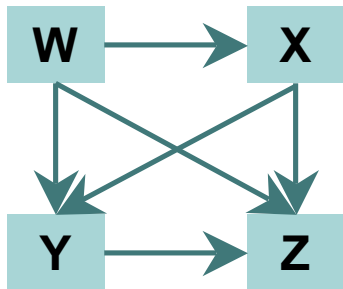
- Announce Sequence of Experiments BEFORE any experiment is performed.



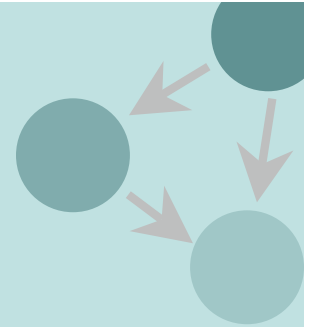
# Example: Strategy A (mult)



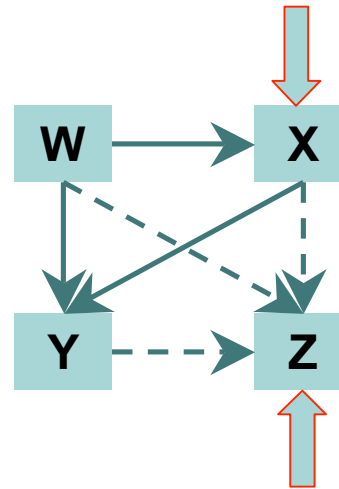
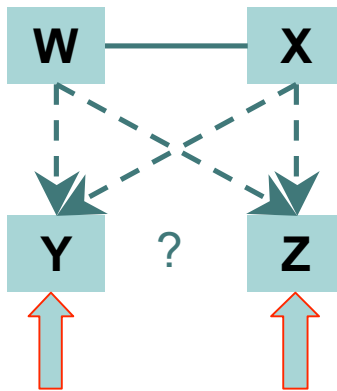
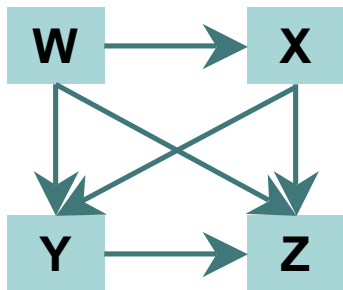
- Announce Sequence of Experiments BEFORE any experiment is performed.



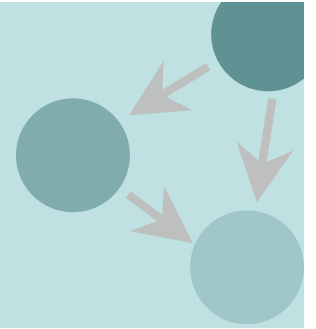
# Example: Strategy A (mult)



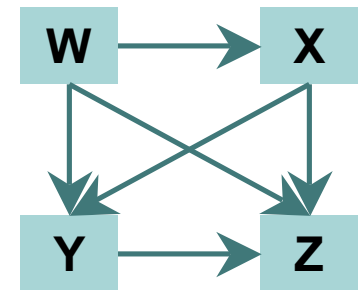
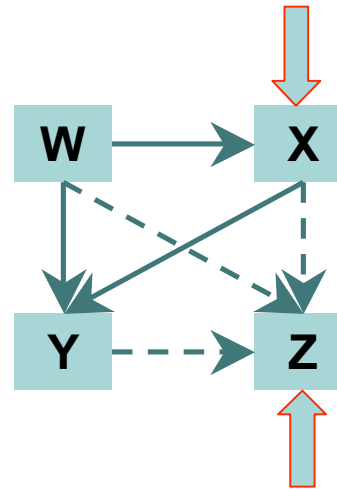
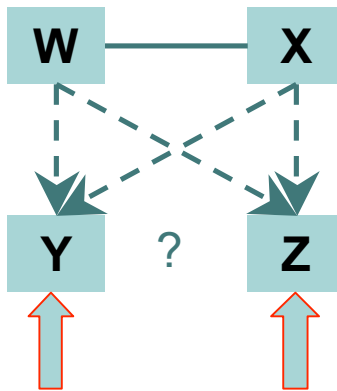
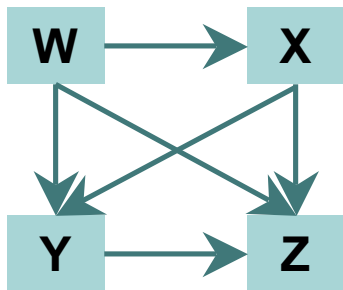
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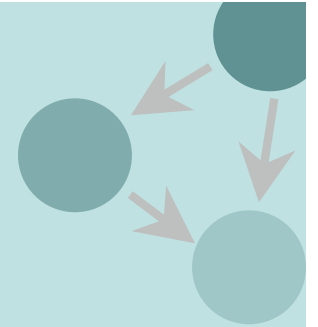
# Example: Strategy A (mult)



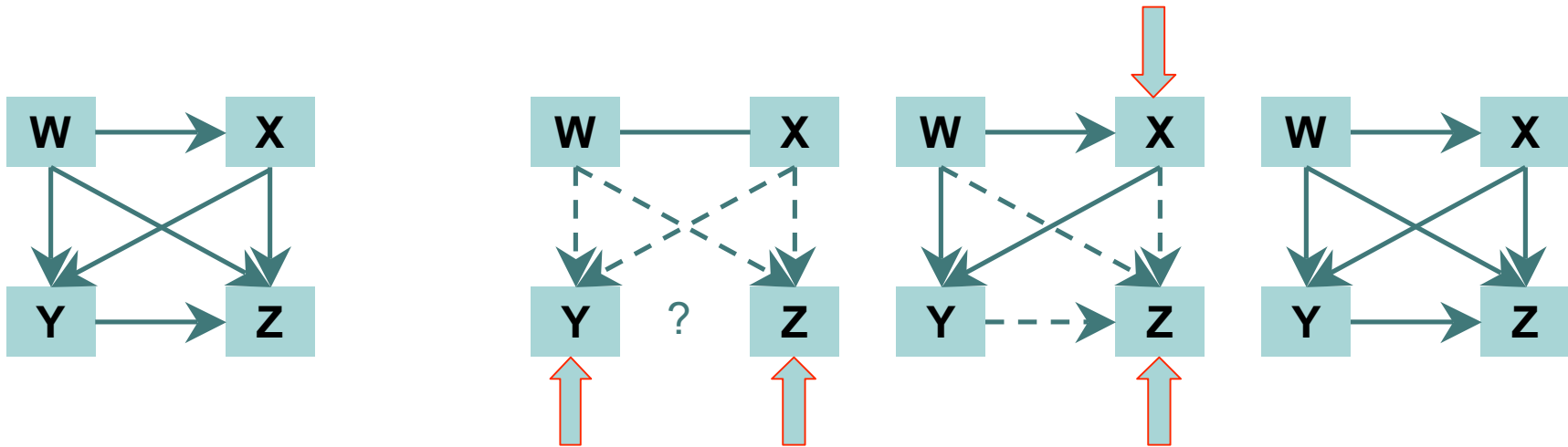
- Announce Sequence of Experiments BEFORE any experiment is performed.



# Example: Strategy A (mult)

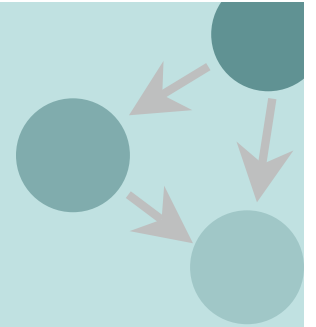


- Announce Sequence of Experiments BEFORE any experiment is performed.



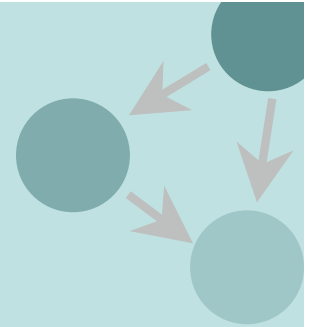
→  $\log_2(N)+1$  experiments are sufficient and in the worst case necessary if multiple simultaneous interventions are permitted per experiment

# Results for Hard Interventions



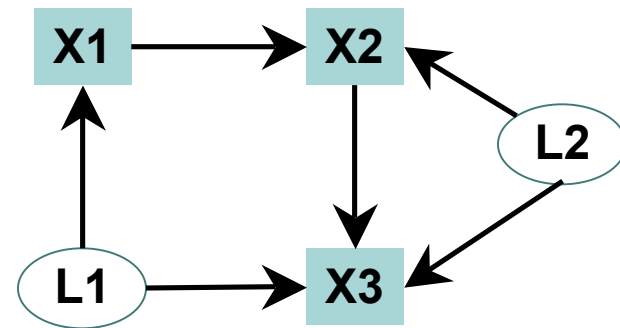
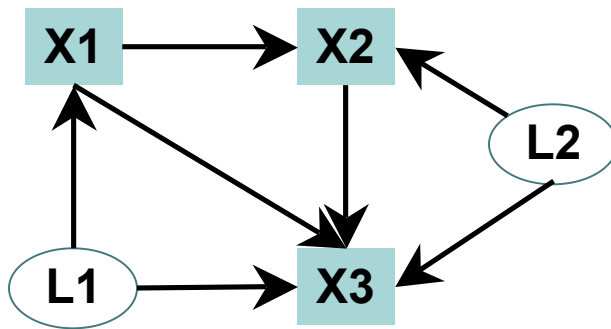
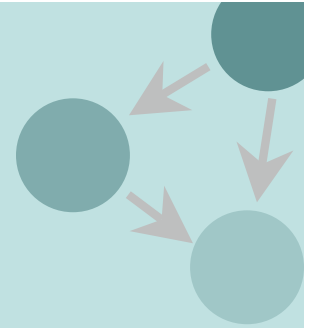
Interventions per experiment	Strength of Intervention	Number of experiments	
Single	Hard	$N-1$	
Multiple	Hard	$\log_2(N)+1$	

# Results for Hard Interventions



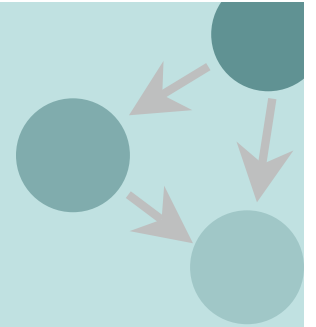
Interventions per experiment	Strength of Intervention	Number of experiments	
		No Latents	Latents
Single	Hard	$N-1$	
Multiple	Hard	$\log_2(N)+1$	

# Latent Variables



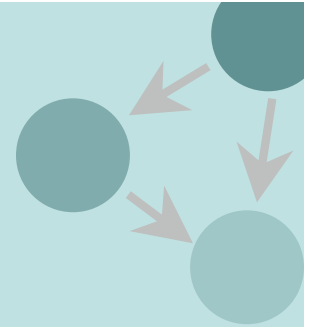
These two graphs are indistinguishable with respect to independence constraints by any sequence of experiments involving single (or no) interventions per experiment.

# Results for Hard Interventions



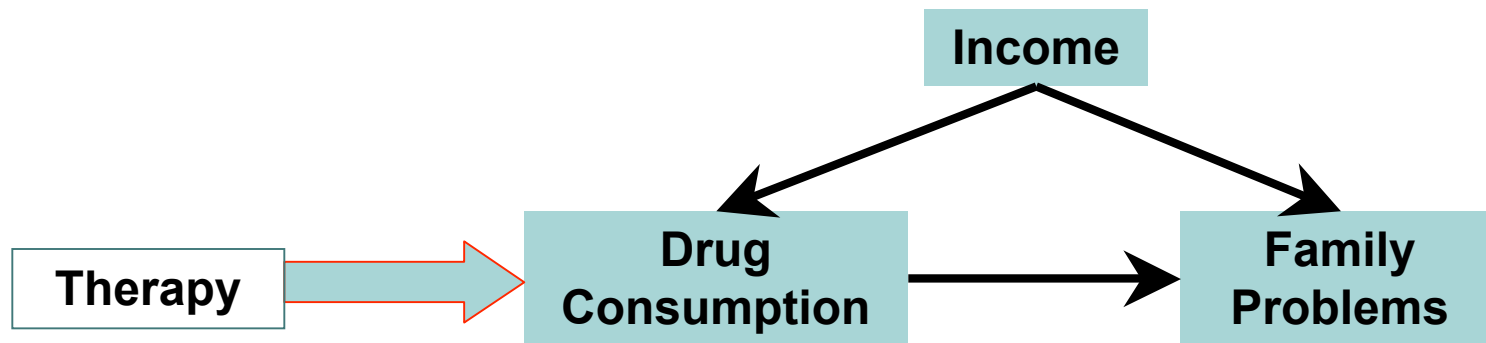
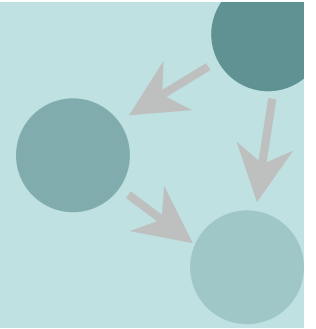
Interventions per experiment	Strength of Intervention	Number of experiments	
		No Latents	Latents
Single	Hard	$N-1$	impossible
Multiple	Hard	$\log_2(N)+1$	

# Results for Hard Interventions

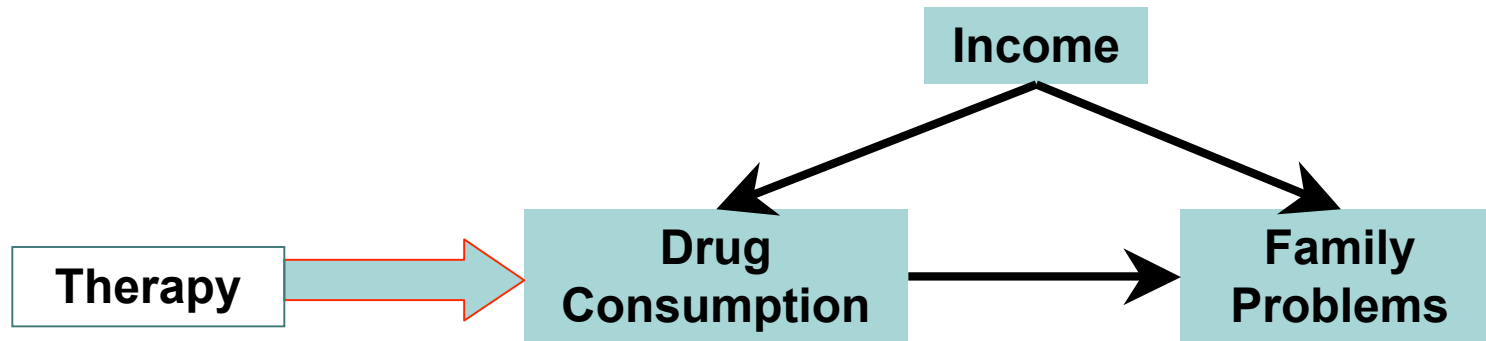
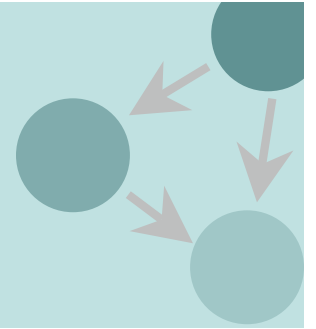


Interventions per experiment	Strength of Intervention	Number of experiments	
		No Latents	Latents
Single	Hard	$N-1$	impossible
Multiple	Hard	$\log_2(N)+1$	$N$

# Soft Interventions

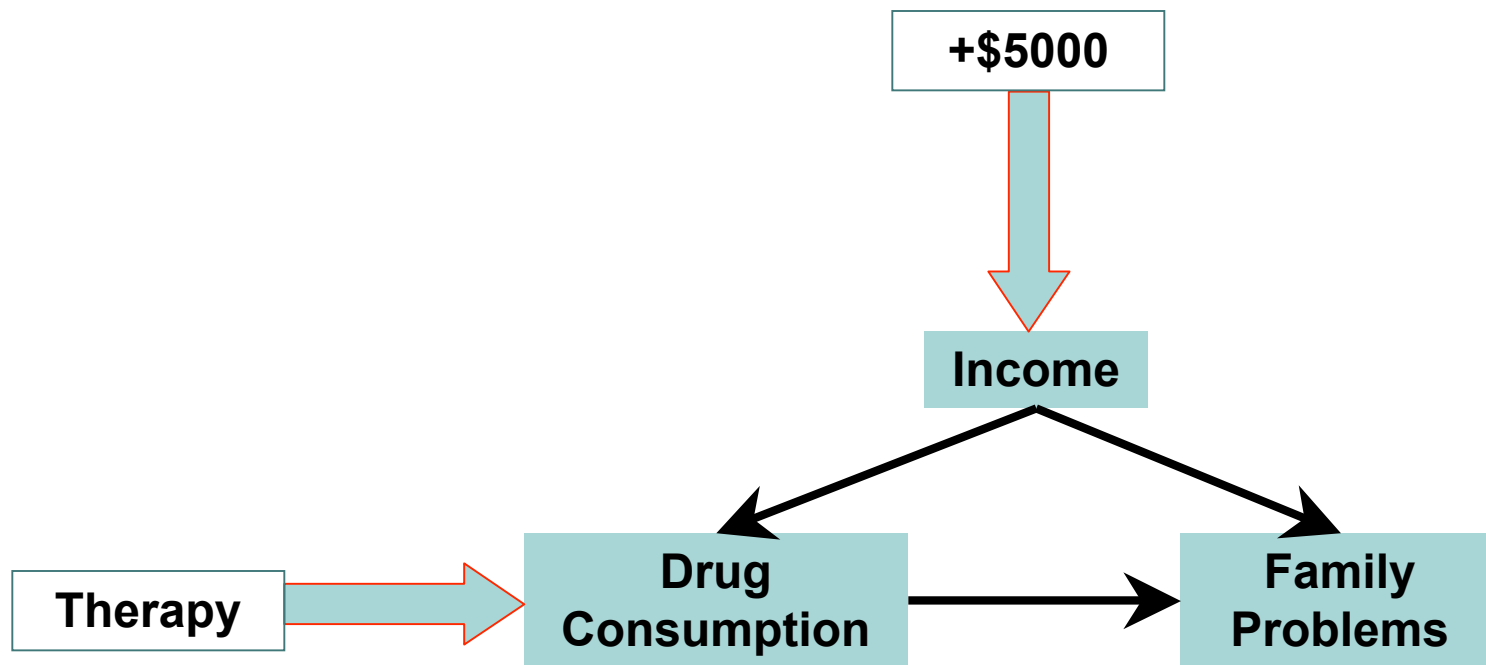
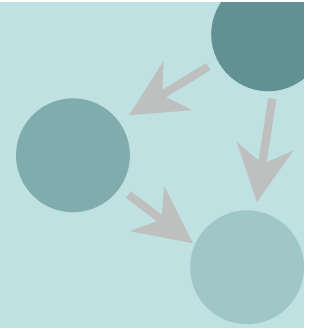


# Soft Interventions



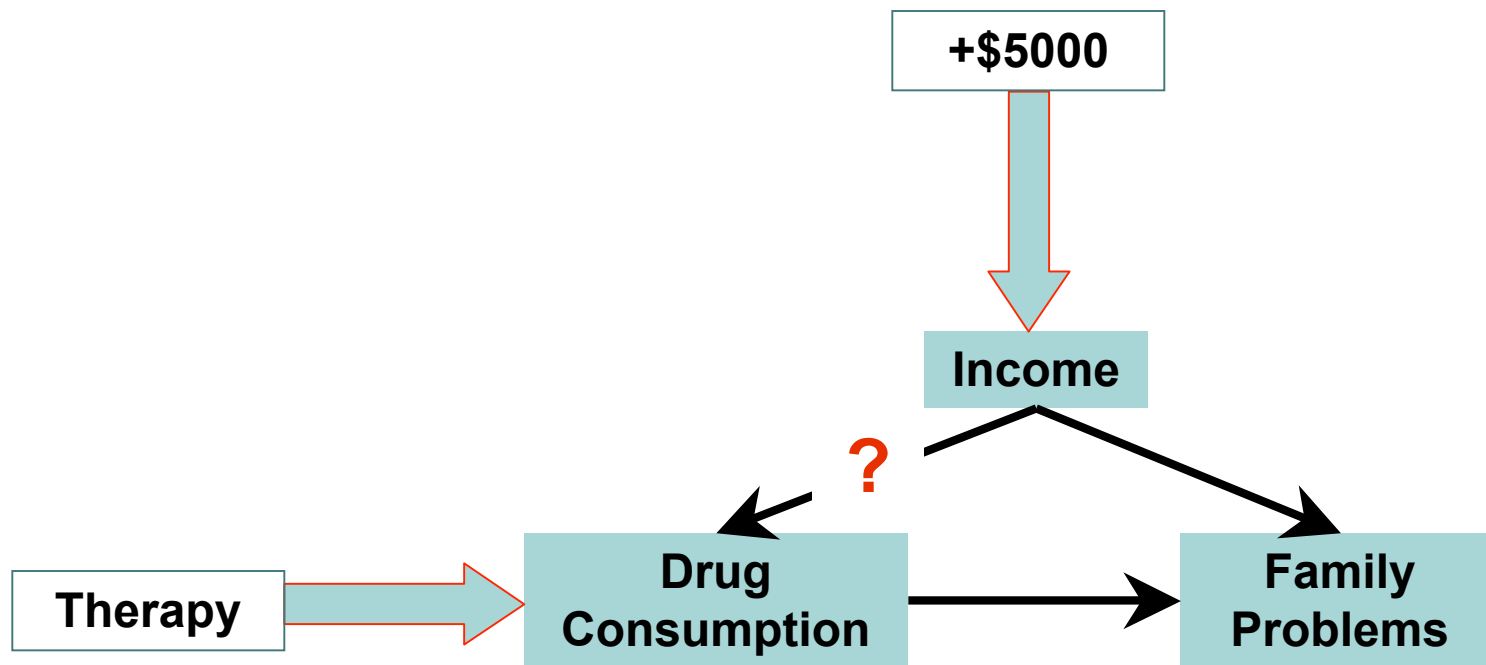
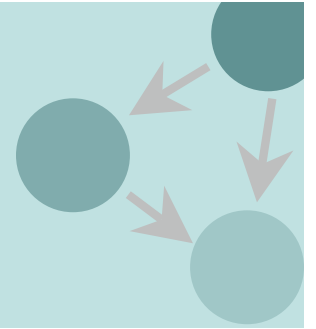
$$P(DC \mid Inc, Th = 0) \neq P(DC \mid Inc, Th = 1) \neq P(DC \mid Th = 1)$$

# Multiple Simultaneous Soft Interventions



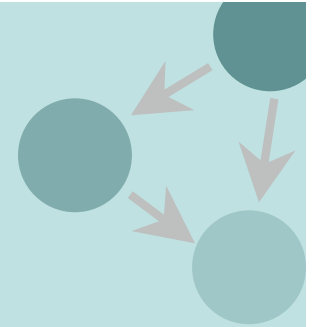
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# Multiple Simultaneous Soft Interventions



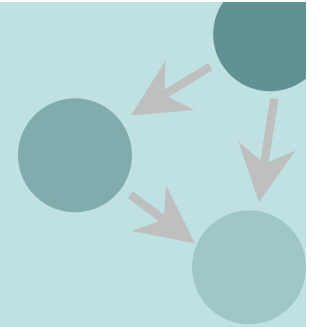
$$P(DC \mid Inc, Th = 0) \neq P(DC \mid Inc, Th = 1) \neq P(DC \mid Th = 1)$$

# Results



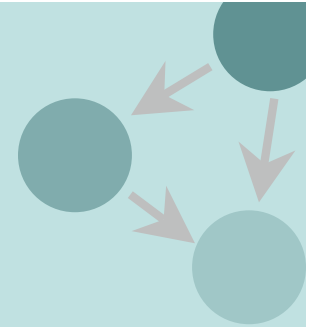
Interventions per experiment	Strength of Intervention	Number of experiments	
		No Latents	Latents
Single	Hard	$N-1$	impossible
Multiple	Hard	$\log_2(N)+1$	$N$
Single	Soft	$N-1$	
Multiple	Soft		

# Results



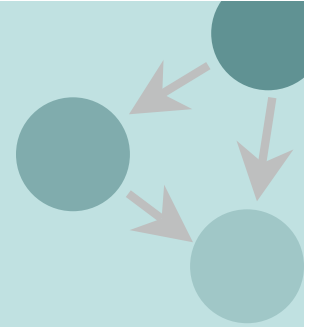
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Multiple	Soft		

# Results



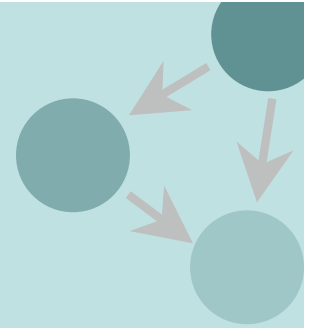
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Multiple	Soft		impossible

# Results



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Single	Soft	$N-1$	impossible
Multiple	Soft	$1!!$	impossible

# Discrete vs. Linear Models



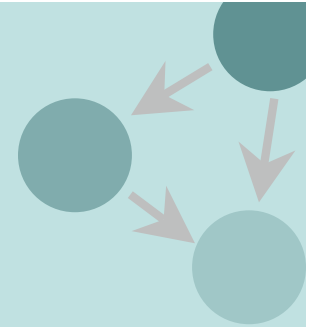
- Discrete

X=1	X=0
0.7	0.3

X	Y=1	Y=0
1	0.8	0.2
0	0.4	0.6

Y	Z=1	Z=0
1	0.1	0.9
0	0.5	0.5

# Discrete vs. Linear Models



## ○ Discrete

X=1	X=0
0.7	0.3

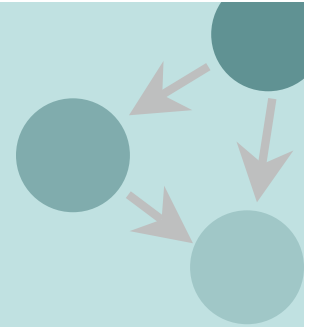
X	Y=1	Y=0
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0	0.4	0.6

Y	Z=1	Z=0
1	0.1	0.9
0	0.5	0.5

## ○ Linear

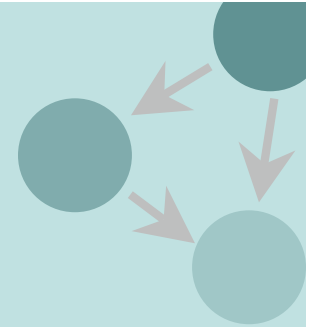
- $X = \varepsilon_1$
- $Y = aX + \varepsilon_2$
- $Z = bY + \varepsilon_3$
  
- cf. structural equation models

# Results for **Linear** Models



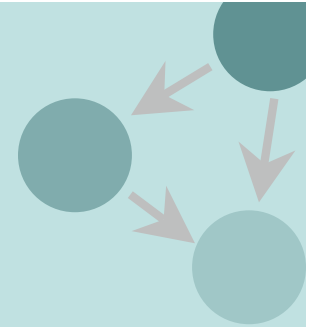
Interventions per experiment	Strength of Intervention	Number of experiments	
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Multiple	Soft	1	impossible

# Results for **Linear** Models



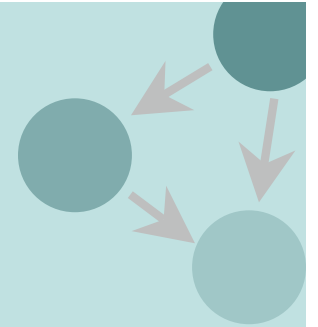
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# Results for **Linear** Models



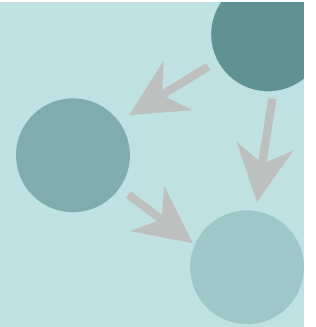
Interventions per experiment	Strength of Intervention	Number of experiments	
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# Results for **Linear** Models



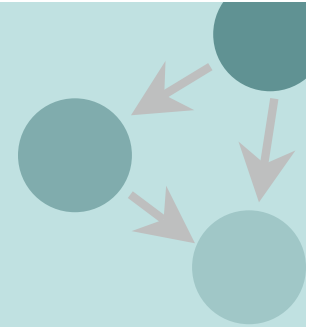
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# Results for **Linear** Models



Interventions per experiment	Strength of Intervention	Number of experiments	
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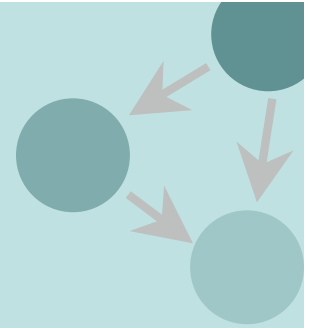
# Results for Linear Models



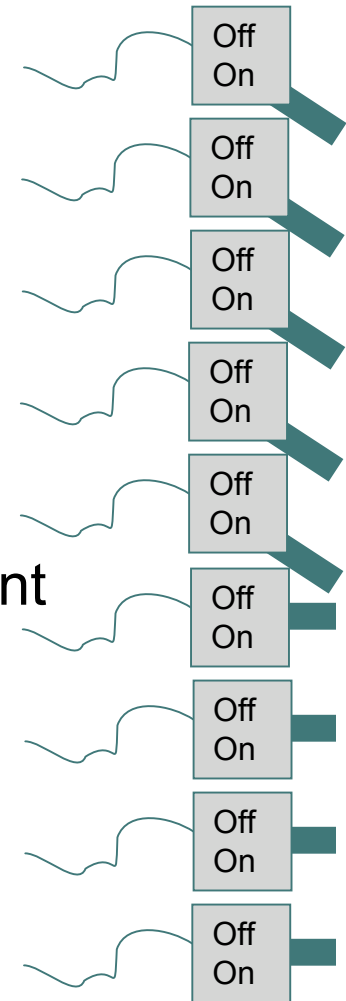
**Bonus:**  
 Presence and location  
 of latent  
 variables and structure  
 among latent variables  
 can be discovered.

Intervention per experiment		Number of experiments	
		Latents	Latents
Single	Hard	$N-1$	$N$
Multiple	Hard	$\log_2(N)+1$	$< 2\log_2(N)$
Single	Soft	$N-1$	$N$
Multiple	Soft	1	1

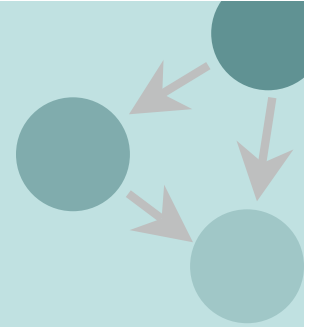
# Assumptions



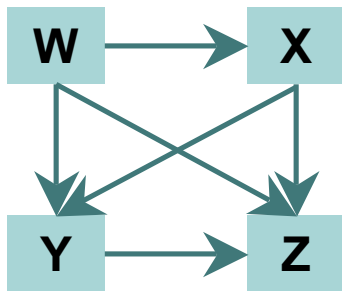
- Causal Markov
- Causal Faithfulness
- Acyclicity
- Interventions possible on every variable
- Independence Oracle
- ➔ ○ Intervention makes intervened variable independent of its normal causes
- ➔ ○ Single Intervention per Experiment
- ➔ ○ Causally sufficient set of variables
- ➔ ○ **Functional Form**



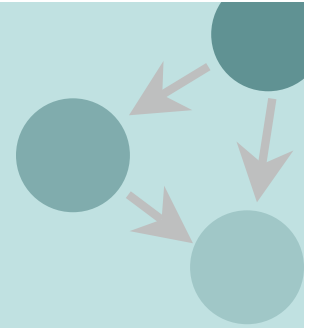
# Adaptive Search Strategy



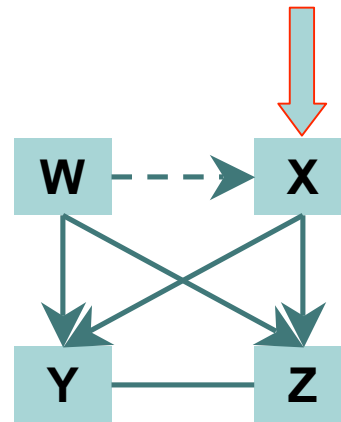
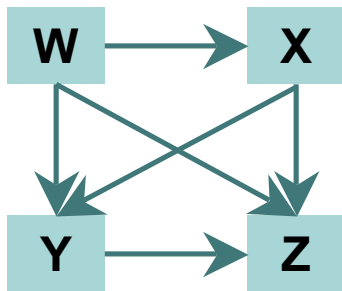
- Specify definite choice of experiment conditional on information obtained so far in the sequence of experiments



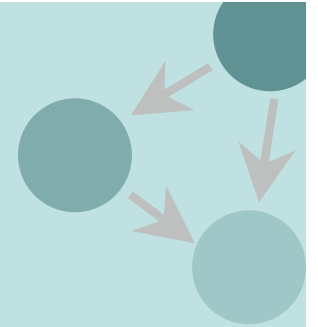
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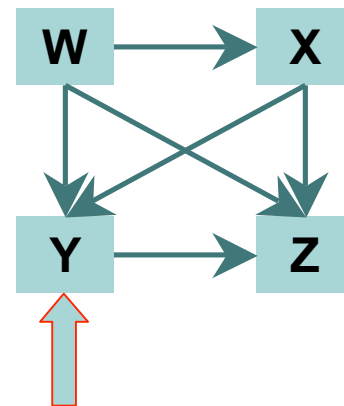
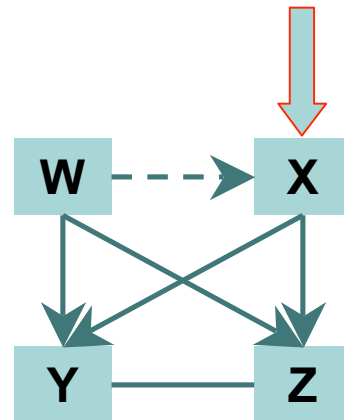
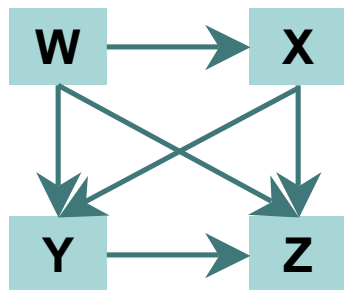
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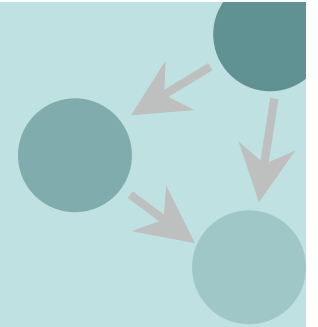
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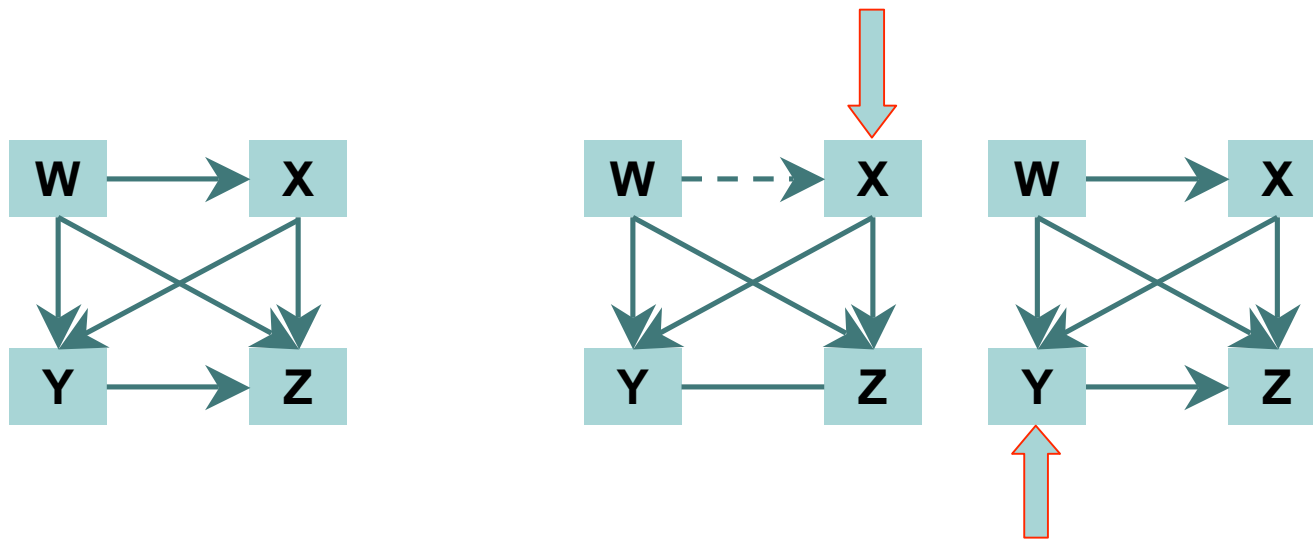
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# Adaptive Search Strategy

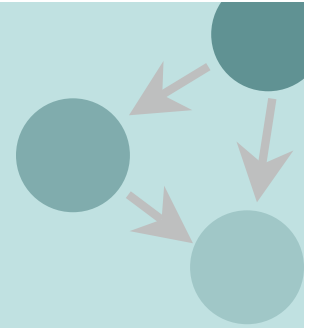


- Specify definite choice of experiment conditional on information obtained so far in the sequence of experiments

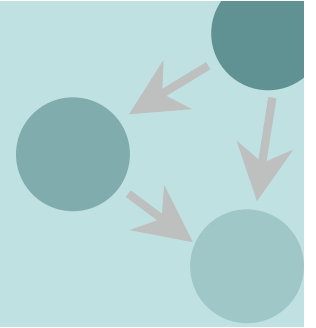


- In general: Worst case is *no better than fixed strategy* results, i.e.  $N-1$  and  $\log_2(N)+1$  experiments, respectively.

# Expected Number of Experiments

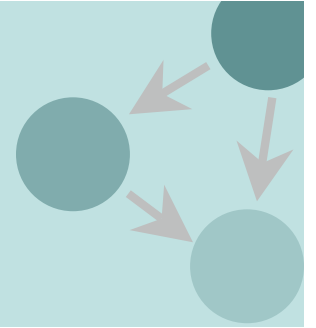


# Expected Number of Experiments



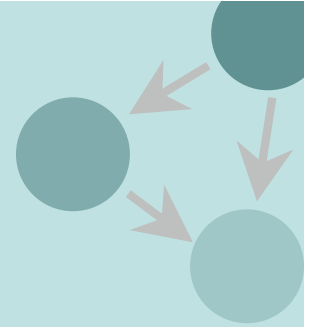
- Expectation with regard to which distribution?

# Expected Number of Experiments



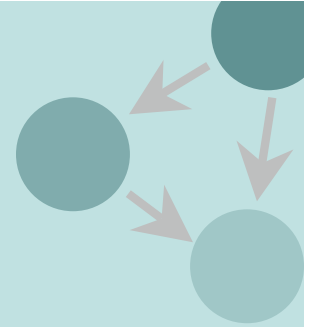
- Expectation with regard to which distribution?
- **Worst Case Expectation:** The maximum of the expected number of experiments for *any* distribution over graphs.

# Search Strategies



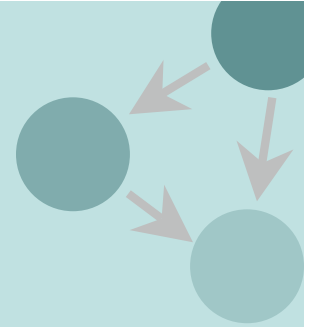
- **Fixed:** Announce the sequence of experiments before any experiment is performed.

# Search Strategies



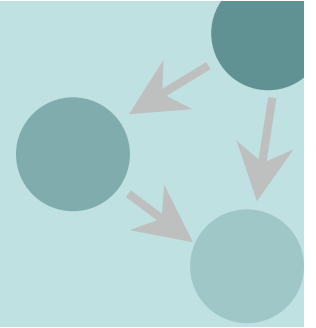
- **Fixed:** Announce the sequence of experiments before any experiment is performed.
- **Adaptive:** Adapt each experiment given the results of previous experiments.

# Search Strategies



- **Fixed:** Announce the sequence of experiments before any experiment is performed.
- **Adaptive:** Adapt each experiment given the results of previous experiments.
- **Mixed:** Use randomization to mix search strategies.

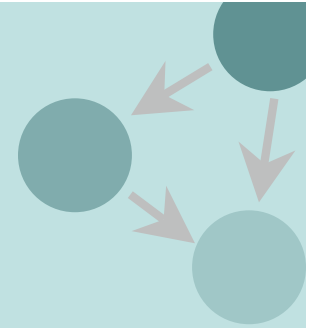
# Search Strategies



- **Fixed:** Announce the sequence of experiments before any experiment is performed.
- **Adaptive:** Adapt each experiment given the results of previous experiments.
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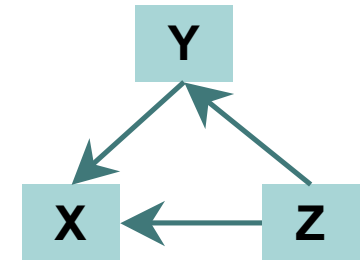
Pure Strategies

# Three Different Strategies



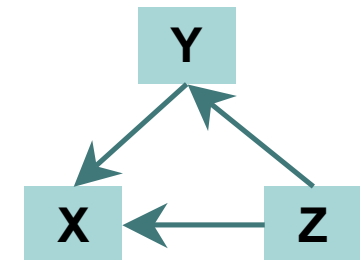
## Fixed Strategy:

Ex1: **X**  
Ex2: **Y**



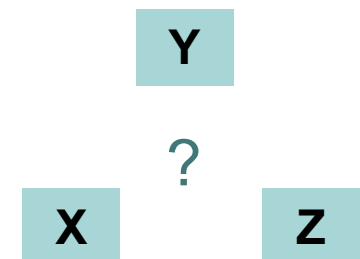
## Adaptive Strategy:

Ex1: **X**  
If resolved, no further experiment, otherwise  
Ex2: **Y**

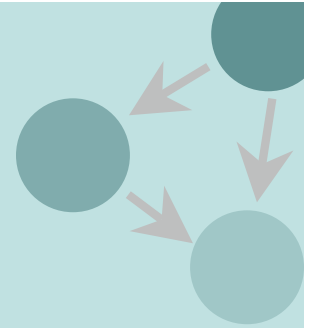


## Mixed Strategy:

Ex1: **K** where K is sampled from  $\{X, Y, Z\}$  with  $1/3$ .  
If resolved, no further experiment, otherwise  
Ex2: **M** where M is sampled with  $1/2$  from  $\{X, Y, Z\} \setminus \{K\}$

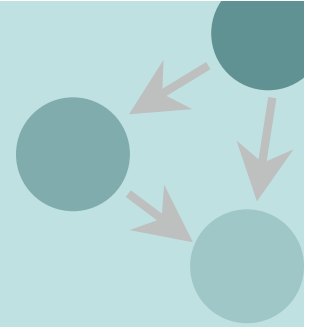


# Results for Strategies



**Theorem 1:** No adaptive search strategy is better than a fixed strategy in the worst case.

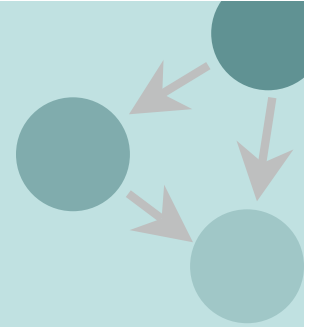
# Results for Strategies



**Theorem 1:** No adaptive search strategy is better than a fixed strategy in the worst case.

**Theorem 2:** Adaptive strategies are better than fixed strategies in expectation for most distributions over graphs.

# Results for Strategies



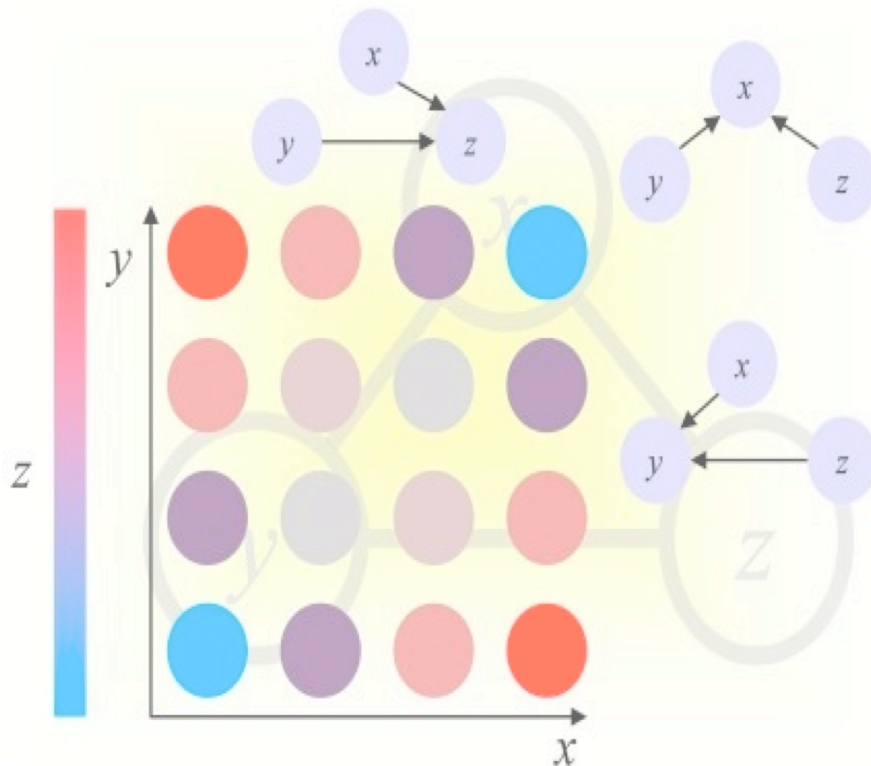
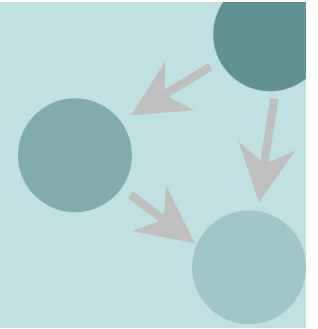
**Theorem 1:** No adaptive search strategy is better than a fixed strategy in the worst case.

**Theorem 2:** Adaptive strategies are better than fixed strategies in expectation for most distributions over graphs.

**Theorem 3:** There is a mixed strategy that strictly dominates any fixed or adaptive strategy in expectation, but only for  $N > 3$  variables.

...

For more on the game theoretic approach (and other topics):

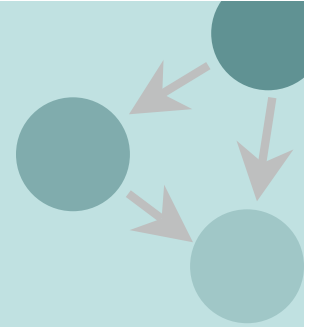


**NIPS 2008 Workshop  
on Causality**

Friday December 12,  
2008

Whistler, CANADA

# Thank You!



Discovery with hard interventions: On the Number of Experiments Sufficient and in the Worst Case Necessary to Identify All Causal Relations Among N Variables (UAI-05, with C. Glymour and R. Scheines)

Discovery with soft interventions: Interventions and Causal Inference (with R. Scheines: Philosophy of Science, 74: 981-995, 2007)

Mixed strategies: Causal Discovery as a Game (NIPS WS 2008)

Optimal experiments given background knowledge: Almost Optimal Intervention Sets for Causal Discovery (UAI-08)

The whole package: Causation and Intervention (PhD Thesis)

## **Frederick Eberhardt**

Institute of Cognitive and Brain Science,  
UC Berkeley;  
Philosophy, Neuroscience & Psychology,  
Washington University in St. Louis  
fde@berkeley.edu