Final-State Interactions in Large-Momentum-Transfer Inelastic Electron-Deuteron Scattering.

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(ricevuto il 25 Agosto 1970)

In the absence of free neutron targets, neutron form factors are normally deduced from electrodisintegration of the deuteron (1). Assuming a one-photon exchange, the impulse approximation away from the forward direction leads to a cross-section for this reaction which is just the sum of the cross-sections of the proton and the neutron, folded into the momentum distribution of the deuteron. This however neglects the effects of final-state interactions between the neutron and the proton. For low momentum transfers, DURAND (2) and McGEE (3) have studied this by making an effective square-well potential model for each of the n-p partial waves. For large energies of the n-p system this procedure is tedious, but in this limit the final state may with advantage be described by an eikonal wave function (4).

If we neglect the effects of spin, then, using the notation of Fig. 1, the amplitude for the scattering from deuterium is

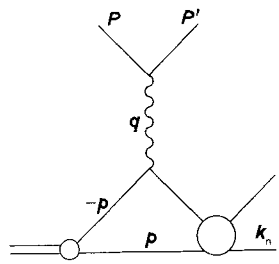

Fig. 1. – Kinematics for the reaction ed→epn. The recoiling proton interacts with the spectator neutron.

$$F = f_p(\boldsymbol{P}, \boldsymbol{q})F(\boldsymbol{k}_n, \boldsymbol{q}) + f_n(\boldsymbol{P}, \boldsymbol{q})F(\boldsymbol{k}_p, \boldsymbol{q}) \,, \tag{1}$$

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(1) W. AUBRECHT, H.-J. BEHRUND, H. DORNER, W. FLAUGER and H. HULTSCHIG: *Phys. Lett.*, **26** B, 642 (1968).
(2) L. DURAND: *Phys. Rev.*, **123**, 1393 (1961); **115**, 1020 (1959).
(3) I. J. McGEE: *Phys. Rev.*, **161**, 1640 (1967).
(4) R. J. GLAUBER: *Lectures at the Summer Institute for Theoretical Physics, Boulder, Colo., 1959*, Vol. **1** (New York, 1960).

where $f_{p(n)}$ is the scattering amplitude from a free proton (neutron) and F is the form factor

$$F(\boldsymbol{k}, \boldsymbol{q}) = \int \psi^*(\boldsymbol{r}) \exp[i\boldsymbol{k}\boldsymbol{r}]\varphi(\boldsymbol{r})\,\mathrm{d}^3r \tag{2}$$

with φ the normalized deuteron wave function and ψ the internal wave function of the recoiling n-p system, which is unity in the absence of rescattering corrections. For large momentum transfers, providing the n-p charge exchange potential is small, the interference term between f_p and f_n is negligible and so we can calculate the cross-sections for scattering from the proton and the neutron separately. The eikonal approximation to the final wave function is

$$\psi^*(\boldsymbol{r}) = \exp\left[-\frac{i}{2K}\int_Z^\infty V_{pn}(\boldsymbol{b}, Z')\,\mathrm{d}Z'\right], \tag{3}$$

where b is the impact parameter for the scattering and Z is the direction of the relative momentum $\boldsymbol{K}$. For momentum transfers $\boldsymbol{q}$, much larger than the Fermi motion, the Z-direction can be taken along $\boldsymbol{q}$. The effective n-p potential V_{pn} can be deduced simply from the n-p amplitude [4], but a more direct approach is preferred. If the range of the n-p force were much larger than the deuteron radius, then we could put $Z=0$ as the lower limit of integration in equation (3) and thus derive a form of Watson's theorem [5]

$$\psi^*(\boldsymbol{r}) = \exp\left[-\frac{i}{2k}\int_0^\infty V(\boldsymbol{b}, Z')\,\mathrm{d}Z'\right] = \exp[i\delta(\boldsymbol{b})], \tag{4}$$

i.e. the final state just acquires a phase given by the impact parameter phase shift $\delta(b)$. In practice we are faced with almost the opposite condition. Because of the hard core, it is a good first approximation to take the neutron and the proton as nonoverlapping in the deuteron, and hence

$$\psi^*(\boldsymbol{r}) = \exp[2i\delta(\boldsymbol{b})]\theta(-Z) + \theta(Z), \tag{5}$$

i.e. if the electron kicks the proton towards the neutron it gets the full phase shift $2\delta(\boldsymbol{b})$, otherwise it stays unchanged. Rearranging (5),

$$\psi^*(r) = 1 - \Gamma(\boldsymbol{b})\theta(-Z), \tag{6}$$

where the impact parameter amplitude $\Gamma(b)$ is connected to the scattering amplitude f

(5) K. M. Watson: *Phys. Rev.*, **88**, 1163 (1952).

of momentum transfer q by

$$\Gamma(\boldsymbol{b}) = \frac{1}{2\pi i K}\int \exp[-i\boldsymbol{q}\cdot\boldsymbol{b}] f(\boldsymbol{q})\,\mathrm{d}^2 q\,. \tag{7}$$

Substituting (7) and (6) into (2)

$$F(\boldsymbol{k}, \boldsymbol{q}) = \varphi(\boldsymbol{k}) + \frac{1}{4\pi^2 K}\int \frac{\mathrm{d}^3 p\,\varphi(\boldsymbol{p})\, f(\boldsymbol{k}^b - \boldsymbol{p}^b)}{k^z - p^z - i\varepsilon}\,, \tag{8}$$

where $\varphi(\boldsymbol{p})$ is the deuteron wave function in momentum space and $\boldsymbol{p}^b$ are the components of $\boldsymbol{p}$ perpendicular to the direction of $\boldsymbol{q}$. The first term in (8) is the usual plane-wave approximation; the rescattering corrections are all in the second term. It is interesting to note that if one writes down a Feynman integral corresponding to Fig. 1 with the « blob » being the p-n amplitude and makes the Glauber approximation of linearizing the propagator of the struck nucleon, one immediately obtains the second term of (8). Unlike most applications of Glauber's theory, the off-shell part of the propagator does not vanish from the final expression [6].

In simple experiments [1] only the angle and momentum of the scattered electron are measured, fixing t and k . Thus we require

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}t\,\mathrm{d}k^z} = \left[|f_n|^2 + |f_p|^2\right] I\,. \tag{9}$$

For high-energy small-angle scattering

$$k^z \simeq \frac{2m(p - |\boldsymbol{p} - \boldsymbol{q}|) + t}{2q}\,. \tag{10}$$

Using our expression (9), and splitting the p^z integrations into principal-part and delta-function contributions, we obtain

$$I = F_1 + F_2^A + F_2^B + F_3^A + F_3^B\,, \tag{11}$$

$$F_1 = \int \mathrm{d}^2 k^o |\varphi(\boldsymbol{k})|^2\,, \tag{12a}$$

$$F_2^A = -\frac{1}{2\pi K}\int \mathrm{d}^2 p^b\,\mathrm{d}^2 k^b\,\varphi^*(\boldsymbol{p}_b, k^z)\,\varphi(\boldsymbol{k}^b, k^z)\,\mathrm{Im}\left(f(\boldsymbol{k}^b - p^b)\right)\,, \tag{12b}$$

$$F_2^B = \frac{1}{2\pi^2 K}\,P\!\int \frac{\mathrm{d}^3 p\,\mathrm{d}^2 k^b}{k^z - p^z}\,\varphi^*(\boldsymbol{p})\,\varphi(\boldsymbol{k})\,\mathrm{Re}\left(f(\boldsymbol{k}^b - p^b)\right)\,, \tag{12c}$$

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[6] D. R. HARRINGTON: *Phys. Rev.*, **184**, 1745 (1969).

$$(13a)\qquad F_3^A = \frac{1}{16\pi^2 K^2}\int d^2p^b\, d^2p'^b\, \varphi(\boldsymbol{p}^b, k^Z)\varphi(\boldsymbol{p}'^b, k^Z) f(\boldsymbol{k}^b - \boldsymbol{p}^b) f^*(\boldsymbol{k}^b - \boldsymbol{p}^b)\, d^2k^b\,.$$

$$(13b)\qquad F_3^B = -\frac{1}{16\pi^4 K^2} PP\int \frac{d^3p\, d^3p'}{(k^Z - p^Z)(k^Z - p'^Z)}\varphi^*(\boldsymbol{p})\varphi(\boldsymbol{p}') f(\boldsymbol{k}^b - \boldsymbol{p}^b) f^*(\boldsymbol{k}^b - \boldsymbol{p}^b)\, d^2k^b\,.$$

Suppose now that the n-p system has insufficient energy for pion production. Then for F_3 we can use unitarity and space inversion

$$(14)\qquad \int f^*(\boldsymbol{k}^b - \boldsymbol{p}'^b) f(\boldsymbol{k}^b - \boldsymbol{p}^b)\, dk^b = 4\pi K\, \mathrm{Im}\,(f(\boldsymbol{p}^b - \boldsymbol{p}'^b))$$

to give

$$(15a)\qquad F_3^A = \tfrac{1}{2} F_2^A\,,$$

$$(15b)\qquad F_3^B = -\frac{1}{4\pi^3 K} PP\int \frac{d^3p\, d^3p'\, \varphi^*(\boldsymbol{p})\varphi(\boldsymbol{p}')}{(k^Z - p^Z)(k^Z - p'^Z)}\, \mathrm{Im}\,(f(\boldsymbol{p}^b - \boldsymbol{p}'^b))\,.$$

Above the pion production threshold, the experiments do not determine whether pions are produced in the rescattering. But F_3 in equation (13) involves only elastic final states. Fortunately the unitarity relation is deficient in exactly the same way, so that if the extra channels are introduced into equation (13) and into the unitarity relation (14) then it can easily be seen that (12) and (15) also describe pion production in the final state. For example they satisfy the closure sum rule, whereas equations (12) and (13) do not, the two neutrons in general not forming a complete set of states.

For computational ease the amplitude was parametrized as a Gaussian

$$(16)\qquad f(q) = i\,\frac{K\sigma_{pn}^T(1 - i\varrho_{pn})}{4\pi}\exp[-\beta^2 q^2/2]\,.$$

The total cross-section data were obtained from the data compilation of WILSON (7), the values of ϱ_{pn} from the dispersion relation calculations of CARTER and BUGG (8) and β^2 was obtained by inspection of the scalar amplitude unfolded from the MacGreggor, Arndt and Wright phase shifts (9) at $t = 0$ and $t = -1$ at each value of K.

The wave function was written as a sum of Gaussians

$$(17)\qquad \varphi(P) = \sum_{i=1}^{5} a_i \exp[-b_i p^2]\,,$$

where the coefficients a_i and b_i were chosen (10) so as to reproduce reasonably the

(7) R. WILSON: *The Nucleon-Nucleon Interaction* (New York, 1963).

(8) A. A. CARTER and D. V. BUGG: *Phys. Lett.*, **20**, 203 (1966).

(9) M. H. MACGREGGOR, R. A. ARNDT and R. M. WRIGHT: *Phys. Rev.*, **182**, 1714 (1968), including references to their earlier work; their data listing: L.R.L. preprint UCRL-50426.

(10) G. ALBERI: private communication.

Gartenhaus s-wave function. The integrals in (13) and (15) can then be done in terms of Gaussians and Dawson's integral D [11]. As a simple example, at the peak of the momentum spectrum of Fig. 2 ($k^Z = 0$, and $F_2^B = F_3^B = 0$) if we take only one term in the wave function expansion (17), we find that the rescattering reduces the cross-section by a factor $(1 - \sigma_{pn}/8\pi b)$, *i.e.* by the order of 4%. This rough estimate is

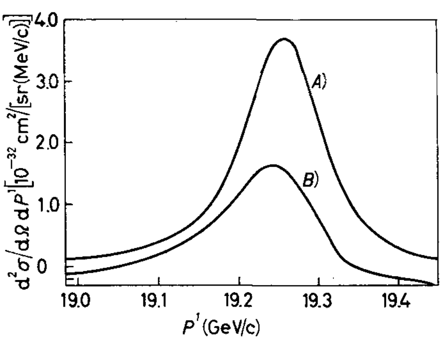

Fig. 2. – Curve A) represents a typical theoretical spectrum for deuteron electrodisintegration. Curve B) is that part of the spectrum which is due to the final-state interaction (a factor of − 10 is included).

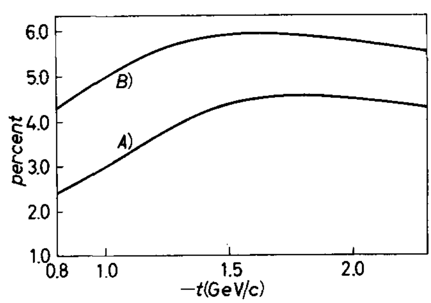

Fig. 3. – Percentage reduction in the peak height due to final-state interactions. Curve A) is calculated via unitarity (eqs. (12) and (15)), while curve B) neglects pion production (eqs. (12) and (13)).

borne out by more detailed calculations with the full wave function. There is a little uncertainty about the values of β^2 to be used, but since $\beta^2 \ll b_i$, this does not significantly affect the results. In Fig. 2 a typical theoretical momentum spectrum of $d\sigma/d\Omega_{lab}\, dP'$ for the electron scattering from the proton only and corresponding to the sum $F_1 + F_2 + F_3$ is plotted against $P' = |\boldsymbol{P} - \boldsymbol{q}|$, the scattered electron's momentum, together with the correction term corresponding to $F_2 + F_3$, for this cross-section. The off-shell contributions, especially F_3^B have a relatively big effect for large k^Z since they decrease only as $1/(k^Z)^2$ rather than exponentially. However the cross-section is then small compared to the background due to direct pion production etc., so that this is not significant. It would though be interesting to look for this tail in coincidence experiments. The reduction factor in the peak height is only a function of t if spin dependence is ignored, and this is plotted in Fig. 3. If the approximate formula (13) rather than (15) is used the effect would be rather larger, up to 7% (see Fig. 3). This is due partly to the pion production and partly to the inability of the parametrization (16) to be unitary.

Work is at present in progress on the inclusion of spin dependence and the deuteron d-state, but preliminary calculations indicate that apart from the well-known apparent reduction in the overall normalization (relative to the s-state calculation) of the order of 5% due to the weak scattering of the d-state [2], the percentage effects of the final-state interaction are not altered by much. It is also hoped to consider these effects

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[11] M. ABRAMOWITZ and I. A. STEGUN: *Handbook of Mathematical Functions* (New York, 1965), p. 298.

in the case of coincidence experiments, which have recently attracted considerable experimental effort [12]. It is possible that the approximate wavefunction (6) or its partial-wave analogue will find other applications in final-state interaction calculations.

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We have benefitted from discussions with Dr. G. ALBERI, Prof. L. CASTILLEJO, Dr. R. J. EDEN, Dr. G. C. FOX, Dr. G. IMMIRZI, Prof. N. STRAUMANN and Mr. R. P. WORDEN. One of the authors (R.S.) acknowledges support from The N.Z. University Grants Committee and Trinity College, Cambridge.

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(12) R. J. BUDNITZ, J. APPEL, L. CARROLL, J. CHEN, J. R. DUNNING jr., M. GOITEIN, K. HANSON, D. IMRIE, C. MISTRETTA, J. K. WALKER and R. WILSON: *Phys. Rev.*, **173**, 1357 (1968); W. BARTEL, F.-W. BÜSSER, W.-R. DIX, R. FELST, D. HARMS, H. KREHBIEL, P. E. KUHLMANN, J. MCELROY, W. SCHMIDT, V. WALTHER and G. WEBER: *Phys. Lett.*, **30** B, 285 (1969).